Emerging Operations Management Problems in the Presence of Competition

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Abstract

Intensified industrial competition, together with fast developments in new technologies, has brought new problems to operations management nowadays. In this thesis, we employ the game-theoretic approach to analyze some of the emerging problems.

In the first study, we investigate the coopetition effect of learning-by-doing which is a common economic phenomenon. We model the problem in a supply chain with two competing OEMs outsourcing to a common CM whose production exhibits the learning-by-doing effect. We find that the learning-by-doing effect intensifies the competition when two OEMs do not have cooperation.. OEMs' total profits under separate learning could be lower than the case with no learning. When OEMs cooperate in learning, we find that the pooled cost reduction could function as a complementary resource which benefits both OEMs. The OEMs' total profits under joint learning are always higher than the case with no learning. Moreover, when the learning speed and the competition intensity are relatively small, the cooperation effect may dominate the competition effect. As a result, the total profits of the OEMs could even increase with the competition intensity. The dominating role of cooperation effect is robust considering the CM's pricing power and various pricing strategies including uniform pricing and myopic pricing. We also find that when the OEMs are differentiated in the market sizes, the structure of a common CM is not stable. The OEM with a much larger market size may prefer to outsource to a separate CM instead.

In the second study, we examine how the downstream competition may influence the usage and effectiveness of trade credit, a commonly used supply chain financing scheme. Specifically, we consider a supply chain in which a supplier sells to two competing retailers and either retailer may be financially distressed. We find that when

financial statuses of the retailers are unbalanced (one retailer relying on the supplier's trade credit and the other on its own capital), the supplier can benefit if the variance of demand shock is moderate. Given the demand uncertainty, the increasing competition intensity may induce the supplier to prefer the balanced retailers' financial status. In addition, for either retailer, the improvement of its competitor's financial capability is favorable if the variance of demand shock is high.

In the third study, we investigate product recovery strategies in a framework of competing supply chains, where two manufacturers sell through their respective retailers. Either manufacturer can choose between two product recovery strategies, collecting used products for remanufacturing by itself (that is, direct recovery) and assigning the task of product recovery to its retailer (indirect recovery). We examine how the competition intensity and the supply chain power structure may influence the equilibrium outcome and its efficiency. Our analysis indicates when the manufacturers and the retailers engage in a vertical Nash game, indirect recovery is the unique equilibrium and is Pareto efficient. However, when the parties engage in a leader-follower game, multiple equilibria occur when the competition intensity is high, and thus either direct recovery or indirect recovery may be chosen.

摘要

日益激烈的行业竞争和日新月异的技术发展为运营管理带来了诸多新的研究问题。在本篇学位论文中，我们运用博弈模型研究其中一些新兴问题。

在第一项研究中，我们探讨了学习曲线的竞合效应。学习曲线是一种常见的经济学现象。我们将模型设定在包含一个共同代工商和两个竞争原始设备制造商的供应链中。代工商的生产表现出学习曲线，这使得原始设备制造商之间的关系从竞争转变为竞合。我们发现当剔除了制造商的合作成分后，学习曲线可能会使得他们的利润比没有学习曲线的情况更低。然而在竞合关系下，我们发现共享的成本削减可以作为有益双方的互补性资源。制造商的利润在此情况下总是高于没有学习曲线的情况。同时我们还发现，当学习速度和竞争程度均较低时，合作效应可能发挥主导作用，导致制造商的总利润随着竞争程度的增加而增加。合作效应占主导作用这一现象在考虑了代工商的定价权和多种定价方式包括一致定价和短视定价的情况下仍旧成立。此外，我们还发现当制造商有不同的市场规模时，共同代工商这一结构并不稳定。拥有较大市场规模的制造商可能会倾向于外包给单独的代工商。

贸易信贷是一种常用的供应链金融合同。在第二项研究中，我们研究下游竞争如何影响贸易信贷的使用和效果。具体而言，我们考虑包含一个供应商和两个竞争零售商的供应链，其中任一零售商都可能存在财务困境。我们发现当市场不确定性处于中等程度时，供应商可能更偏好非平衡的下游财务状况（一个零售商依靠供应商提供的贸易信贷，另一个零售商依靠自有资金）。当给定市场不确定性，竞争程度的增加可能使得供应商偏好对称的下游财务状况。对于任一零售商，竞争者财务状况的提升在市场不确定性较高时是有利的。

在第三项研究中，我们讨论了竞争供应链中的产品回收问题。在两条竞争供应链中，生产商通过各自的零售商销售。对生产商而言，存在两个产品回收策略选择：自行回收产品（直接回收）和通过零售商回收（间接回收）。我们探究了竞争

程度和供应链权力结构如何影响均衡结果及其效率。分析表明，当生产商和零售商处于纳什博弈时，间接回收是唯一均衡并且是帕累托最优。然而，如果生产商和零售商处于序贯博弈时，当竞争程度高时可能会出现多种均衡，即直接回收和间接回收均可作为均衡策略。

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Chapter 1

Introduction

In the wake of the new economic development, competition becomes a crucial consideration for the operations decisions of the firms. The operations strategies should be adjusted to accommodate the competition. On the other hand, the competition also changes the implications of many operations problems. Many interesting research questions arise in the new background. This thesis studies three emerging operations problems in the presence of competition, including learning-by-doing effect, trade credit, and product recovery.

Learning-by-doing is a prevailing economic phenomenon in which the production cost decreases with the production quantity. It has been widely observed in many industries ever since it was first observed by Wright (1936) in the airframes industry. Learning-by-doing is also an important competition consideration as pointed out by Jarmin (1984) which studies the early rayon industry of U.S. While outsourcing becomes a standard manufacturing option, competing original equipment manufacturers (OEMs) may rely on a common contract manufacturer (CM) for production in many cases. For example, Foxcoon produces for both Sony Playstation and Windows Xbox. The learning-by-doing effect of the CM changes the pure competition relationship between the OEMs to coopetition. The OEMs are competing in final market while cooperating to achieve the cost reduction by enhancing the CM's learning effect.

In the first study, we investigate the coopetition effect of learning-by-doing in a supply chain with two competing OEMs outsourcing to a common CM whose production exhibits the learning-by-doing effect. First of all, we find that the learning-by-doing effect intensifies the competition when we exclude the cooperation between OEMs. OEMs' total profits under separate learning could be lower than the case with no learning. When OEMs cooperate in learning, we find that the pooled cost reduction could function as a complementary resource which benefits both OEMs. The OEMs' total profits under joint learning are always higher than the case with no learning. Moreover, when the learning speed and the competition intensity are relatively small, the cooperation effect may dominate the competition effect. As a result, the total profits of the OEMs could even increase with the competition intensity. The dominating role of cooperation effect is robust considering the CM's pricing power and various pricing strategies including uniform pricing and myopic pricing. We also find that when the OEMs are differentiated in the market sizes, the structure of a common CM may be unstable. The OEM with a much larger market size may prefer to outsource to a separate CM instead.

Trade credit is a commonly used contract in supply chain finance. It constitutes a major source of firm's short-term financing (Petersen and Rajan 1997). Compared to the professional financial intermediates, suppliers hold many merits to provide the credit to retailers such as a lower transaction cost (Emery 1984), tax saving (Brick and Fung 1984), etc. Researchers also identify the operational effect of trade credit, such as sharing the demand risk (Yang and Birge 2017), coordinating the supply chain (Xiao et al. 2016, Lee and Rhee 2011). In addition the effect in the vertical relationship, trade credit can also soften the horizontal competition as observed by Peura et al. (2017).

However, how trade credit influences the joint vertical and horizontal relationship is not clear.

In the second study, we examine the usage and effectiveness of trade credit with downstream competition. Specifically, we consider a supply chain in which a supplier sells to two competing retailers and either retailer may be financially distressed. The trade credit changes the competing behaviors of the supply chain members. First of all, we find that the supplier may bail out the financially distressed retailer when it is competing with a retailer who has sufficient capital, compared to the case where two financially distressed retailers compete with each other. When both retailers are financially distressed, retailers sells more than the benchmark where both retailers have sufficient capital when the variance of demand shock is relatively large, indicating a more intensified downstream competition. The retailer with sufficient capital may sell more to predate the financially distressed retailer when the variance of demand shock is in moderate. From the perspective of profitability, the supplier can benefit from providing the trade credit, demonstrating the prevalence of trade credit. Compared to the case where both retailers are financially distressed, the supplier may prefer the case where retailers have unbalanced financial statuses (one retailer relying on the supplier's trade credit and the other on its own capital) when the variance of demand shock is moderate. The supplier may prefer the case where both retailers are financially distressed when the variance of demand shock is relatively large. Retailers' profits are enhanced with better financial statuses. However, the improvement of its competitor's financial status is favorable only when the variance of demand shock is relatively high.

Remanufacturing shows great economic potential and strategic importance in competition. Product recovery is a fundamental step of the remanfacturing system. In the third study, we investigate how the supply chain competition and power structure influence the product recovery strategies. We model the problem in two competing supply chains, where two manufacturers sell through their respective retailers. Either manufacturer can choose between two product recovery strategies, collecting used products for remanufacturing by itself (that is, direct recovery) and assigning the task of product recovery to its retailer (indirect recovery). The manufacturer and retailer in either supply chain are engaged in three types of game sequence: Stackelberg-manufacturer as the leader, Stackelberg-retailer as the leader, and vertical Nash. Our analysis indicates when the manufacturers and the retailers engage in a vertical Nash game, indirect recovery is the unique equilibrium and is Pareto efficient. However, when the parties engage in a Stackelberg leader-follower game, multiple equilibrium occur when the competition intensity is high, and thus either direct recovery or indirect recovery may be chosen. The channel power determines either direct recovery or indirect recovery as the low-price strategy, because the firm with leadership power is less effective to collect the used products due to the issue of double marginalization. Manufacturers may be trapped into prisoners' dilemma for choosing the low-price strategy.

Chapter 2

The Coopetition Effect of Learning-by-Doing

2.1 Introduction

Learning-by-doing is a well-known economic concept which relates the productivity growth to the accumulation of production experience by firms. Specifically, the production cost declines with the cumulative production quantity through the accumulated experience from repetition of work, a passive learning process instead of positively investing capital or adding labor. The phenomenon of learning-by-doing is prevailing in various industries, including airframes (Wright 1936), machine manufacturing (Baloff 1971) and semiconductors (Webbink 1977). Among others, the Japanese automobile manufacturer Toyota is famous for the management philosophy requiring continuous improvement, known as *Kaizen*, which is explicitly built upon learning-by-doing effect (Shingo 1981). Learning-by-doing effect is also an important consideration for firms' competing strategies. For example, Jarmin (1984) shows that, in the early rayon industry of US, firms consider the strategic implication of learning-by-doing from competitor when making their output decisions.

With the development of global supply chain management, more and more Original Equipment Manufacturers (OEMs) choose to outsource the production to the Contract Manufacturers (CMs) for different reasons. The CMs are growing at an amazing speed. In the electronics industry, the top six CMs grew at a rate of 43% per year from 1995 to 2002 (Sturgeon 2002). In the future, CMs in this industry are expected to grow at double digit spead beyond the development of the whole industry (Jorgensen 2006). The fast development also implies the effect of learning-by-doing since the CMs could get more experiences from practices (Shih 2018). Moreover, the CMs are leading a revolution in the trend of smart factory for the advantage on the learning curve (Grylls 2018). Foxconn functions as a good example of CM who stands out in learning curve and manages to develop the business with tight margin on the supply chain (Yang 2018, White 2017).

The development of contract manufacturing industry also concentrates to several big companies. In many cases, OEMs may depend on common CMs for production. For example, BYD, a Shenzhen-based contract manufacturer, produces lithium batteries for OEM customers such as Motorola and Nokia (Sodhi and Tang 2012). Haier, a consumer electronics giant, used to be contract manufacturer of refrigerators for LG and Samsung.

The learning curve of the contract manufacture is surely influencing OEMs' competing strategies especially when the OEMs share a common CM. The relationship of the OEMs changes from pure competition to the coopetition paradigm. Since they share the benefits of learning-by-doing effect, they cooperate in enhancing the CM's learning effect by setting larger production quantities but in the meanwhile they're at a competition edge. Take the video game industry as an example. From 1998 to 2001, the industry became the fastest growing segment of the entertainment industry with a growth rate of 15 to 25 per cent. Sony Playstation and Microsoft Xbox are two leading firms in the industry. Foxconn, the world's largest contract electronics manufacture, produces for both Sony Playstation and Microsoft Xbox. These product lines used to bring large amount of profit to Sony and Microsoft. But with the accelerating of competition, both of them are selling at tight margin as Wood (2013) identifying from the price teardown. The industry analyst believe that both Sony and Microsoft will benefit from the cost reduction "according to the normal learning curve dynamics". By the end of year 2016, Playstation 4 cut price to $250. Xbox one soon took action to respond Sony's price cut. "It's matching the price of Sony's console as it continues to try and compete in the market against the PlayStation 4." (Farooqui 2017)

To this end, we would like to investigate the following research problems: what's the influence of learning-by-doing on the competing OEMs' prices and profits when they outsource to a common CM? How does the change of competition intensity influence the competition effect and cooperation effect? What're the effect of other pricing strategies, such as uniform pricing and myopic pricing? What's the effect of channel power structure? What's the competing OEMs' incentive to engage in the joint learning?

To address these problems, we consider a two-period model in which two competing OEMs depend a common CM for manufacturing the products. OEMs are engaged in Bertrand competition in either period. CM's production embodies the learning-by-doing effect, that is, the unit production cost in the second period is reduced from the first-period production cost. The reduction of CM's production cost is linear in the total quantity for two OEMs in the first period. Hence, the relationship between the OEMs changes from pure competition to coopetition paradigm. They're cooperating to get more cost reduction in the second period while competing in the final market. To help isolate the competition effect from the cooperation effect, we also consider an auxiliary model in which two OEMs outsource to separate CMs.

The learning-by-doing effect has strategic impact on the competition behavior of OEMs. In the first period, OEMs price lower than the case without learning to enjoy the cost advantage from more significant learning. The profit loss in the first period could be viewed as the expense to exploit larger cost reduction from learning. When OEMs outsource to separate CMs, the expense might be so large that the gaining in the second period is insufficient to compensate. Under separate learning, the OEMs may have lower total profit than the case without learning. When OEMs outsource to a common CM, the pooled cost reduction functions as a complementary resource which benefits both OEMs. The benefit from the second period is always sufficient to compensate for the expense in the first period. The joint learning is beneficial to both OEMs and consumers since OEMs can obtain higher profit and consumers can get lower prices.

When OEMs outsource to separate CMs, the learning-by-doing effect intensifies competition. With the increasing of competition intensity, the OEMs' total profit is decreasing at a higher speed than the case without learning. When OEMs outsource to a common CM, the role of the increasing competition intensity is two-fold. It could, on one hand, facilitate the cooperation in cost reduction. With the increasing of competition intensity, the total profit of OEMs firstly increases and then decreases, reflecting the tangling of competition effect and cooperation effect. When the total profit is increasing with competition intensity, the cooperation effect is dominating the competition effect. The result is robust considering the CM's pricing power and different pricing strategies including uninform pricing and myopic pricing.

OEMs would prefer to outsource to a common CM when they have symmetric market since the total profit under joint learning is always higher than separate learning. But when OEMs are asymmetric in the market size, the OEM with larger market size may prefer to outsource to different OEMs. Because the OEM with larger market size contribute large proportion to the cost reduction, giving the OEM with smaller market size a free ride.

The remainder of this study is organized as follows. In section 2.2, we provide a review on the literature of learning-by-doing. Model formulation and the analysis for base model are provided in Section 2.3. In Section 2.4, we explore several related directions that complete our understanding about the learning-by-doing effect. Section 2.5 summarizes the conclusions and gives the future research directions.

2.2 Literature Review

Our research is firstly related to the literature on learning-by-doing. The concept of learning-by-doing refers to the decrease of unit production cost resulted from the accumulated production experience. It is also depicted as learning curve in the context of manufacturing. The phenomenon of learning-by-doing has long received attention by researchers since the pioneering work of Wright (1936). It is found that the direct labor cost decreases by 20% with every doubling of cumulative quantity manufactured in the areospace industry. As documented in Yelle (1979), various empirical researches have devoted to identify and estimate the effect of learning-by-doing in different industries, such as machine manufacturing (Baloff 1971) and semiconductors (Webbink 1977).

Adler and Clark (1991) further identify a hierarchical mechanism that underpin the learning-by-doing effect: the first-order learning comes from the continuous repetition work experience while the second-order learning relates to the engineering process modifications happen at discrete intervals. As a result, although learning accrues continuously, the application of accumulated learning is often periodic. Correspondingly, in many analytical research, such as Fudenberg and Tirole (1983), Dasgupta and Stiglits

(1988) and Jin et al. (2004), the production cost reduction does not happen within one period while the learning accumulates continuously. The production cost reduction is related to the quantity of earlier production batch instead.

The analytical research about learning-by-doing dates back to the seminal paper of Arrow (1962). Some researchers study the macroeconomic effects of learning-by-doing (e.g., Romer 1986, Young 1991). The existence of learning-by-doing effect could be also used to explain many operations problems. For example, in the 1980s, the US manufacturers are outweighted by the Japanese competitors for both quality and cost. The usual understanding of the trade-off between quality and cost is challenged. Fine (1986) resolves the controversy by introducing quality-based learning curve.

On the other hand, the analytical research that focuses on the application of learning-by-doing effect to operations management problem including optimal production planning (Mazzola and McCardle 1997), cost reduction investments (Fine and Porteus 1989, Bernstein and Kök 2009), capacity expansion (Hiller and Sharpiro 1986), product quality (Li and Rajagopalan 1998) and sourcing decision (Silbermayr and Minner 2016). This round of attention towards learning-by-doing effect partially results from the extraordinary performance of the Japanese companies who are famous for the philosophy of continuous improvement, which explicitly depends on the learning-by-doing effect (Shingo 1981).

In addition to the application of learning-by-doing in the centralized decision making, it also influences the interaction in one-to-one supply chain relationship. Gray et al. (2009) consider a two-period game between an OEM and a powerful CM wherein both firms can reduce their production costs through learning-by-doing. The learning-by-doing effect lead to dynamic outsourcing in which the make-buy choice should change from one period to the next because the OEM and CM production evolve based on past production levels. Li et al. (2015) study the issue of supply chain coordination in a decentralized two-echelon supply chain where a manufacturer produces with the benefits of learning-by-doing. It shows that the double marginalization problem becomes more severe with the increasing of mean learning rate and the learning rate variability.

Several economics papers have considered the problem of learning-by-doing effect and horizontal competition. Fudenberg and Tirole (1983) show in a two-period Cournot model that firms may choose decreasing output paths considering the strategic effect of learning on the competitors. Learning is socially beneficial. Spence (1981) points out that learning curve could create entry barrier and protection from competition. Adopting different models, Dasgupta and Stiglitz (1988) and Carbral and Riordan (1994) focus on how learning influences the market structure evolution. Using the firm-level data from American rayon industry, Jarmin (1984) suggests that the firm does consider the strategic implication of learning-by-doing from competitor when deciding the output decisions. But these papers only consider the case when manufacturers learn individually. We consider a supply chain structure where the competing manufacturers are depending on a common CM for production and thereby share the joint learning-by-doing effect.

To the best of our knowledge, this research is the first to investigate the coopetition effect of learning-by-doing. Therefore, our work is also related to the general research area on coopetition. Coopetition implies the co-existence of competition and cooperation between two or multiple related parties who simultaneously pursue individual goals (Nalebuff and Brandenburger 1996). Research about the coopetition in supply chain is not rare (Bakshi and Kleindorfer 2009, Gurnani et al. 2007, Sodhi and Tang 2013). In operations management, common activities which induce cooperation among competitors include group buying (Keskinocak and Savaşaneril 2008, Chen and Roma 2011), capacity sharing (Li and Zhang 2015), subcontracting (Xu et al. 2017) etc. Among the above research about coopetition, we didn't find study about the

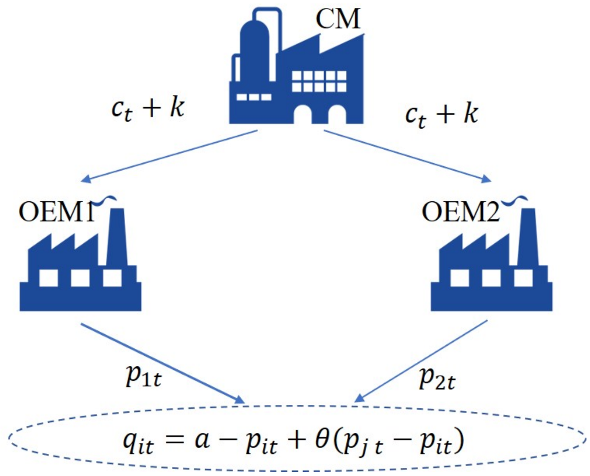


Figure 2.1: Supply chain structure

learning-by-doing effect. In this paper, we find that the learning-by-doing has coopetition effect. The cost advantage induced by the learning-by-doing could function as a complementary resource, naturally forming cooperation relationship between competitors. Under this structure, many results are contrary to our traditional understanding about the competing behaviors of OEMs.

2.3 Model

2.3.1 Base Model

We consider a two-period ($t \in \{1, 2\}$) model to capture the intertemporal learning-by-doing effect in the following supply chain: two competing original equipment manufacturers (OEMs) who rely on one common contract manufacturer (CM) for manufacturing. We use the subscripts i ($i \in \{1, 2\}$) and $j = 3 - i$ to represent the two OEMs. This exactly corresponds to the structure of Sony, Microsoft and Foxconn.

The OEMs are engaging in Bertrand competition at both periods. The OEM i's demand, q_{it}, decreases with its own price p_{it}, and increases with the opponent's price

p_{jt}. Demand function is given by $q_{it} = a - p_{it} + \theta(p_{jt} - p_{it})$, where $a > 0$ is the market potential, and $\theta \geq 0$ is the competition intensity which measures the substitutability between two OEMs. With higher substitutability, the competition between the OEMs is more intensive. We assume that the OEMs have steady and symmetric potential market in the base model. This formulation can also be seen in Chen and Roma (2011) and Tsay and Agrawal (2000). Throughout the paper, we use the subscript "*it*" to represent the decision of OEM i in period t. We do not consider the issue of inventory carryover. Manufacturers who adopt lean production system would not produce more than what they sell (Gray et al. 2009).

CM's production cost exhibits the effect of learning-by-doing. The more the CM produces, the more experience it accumulates thereby the unit production cost is lowered. We adopt the concept of *batch learning*: although the learning is continuous, the cost reduction only happens in the new production batch. To be specific, in the first period, the CM has a constant marginal cost $c_1 = c$. In the second period, the CM's unit production cost depends on the total first-period production quantity: $c_2 = c - \lambda(q_{11} + q_{21})$, where q_{11} and q_{21} are quantities outsourced by two OEMs in the first period, and $\lambda \geq 0$ measures the CM's learning speed. This linear learning function has been used in literature like Jin et al. (2004), Hiller, Shapiro (1986) and Shum et al. (2016). Although there're various learning models as summarized in Yelle (1979), this linear formulation is a good approximation of the true learning process (Carlson 1961). One underneath driver for the learning-by-doing effect is the increasing productivity of the labor with the accumulation of experience. Wright (1936) observed that the direct labor hours of producing one unit of product decreases at a uniform rate with the quantity of units manufactured doubling. This uniform rate is reflected as the linear decreasing format of unit production cost. The larger the uniform rate, the steeper slope of the learning curve is.

We assume that the CM charges a fixed premium k to OEMs in both periods. That is, the unit wholesale price for OEMs is $c_1 + k$ in the first period, and $c_2 + k$ in the second period. The passive role of contract manufacturer is not rare in literature, such as Anderson and Parker (2002). This is also common in practice since the contract manufacturer is usually lack of pricing power in the supply chain. The pressure in the CM industry is so fierce that each firm functions as a price taker. For example, Foxconn charges a mere \$8 markup for per iphone. From the price teardown of Playstation 4 and Xbox One, we find that the cost of final assembly and test are \$17 and \$16 respectively, which could be viewed as the markup of the contract manufacturer. Furthermore, we also assume the markup k as common knowledge in the industry.

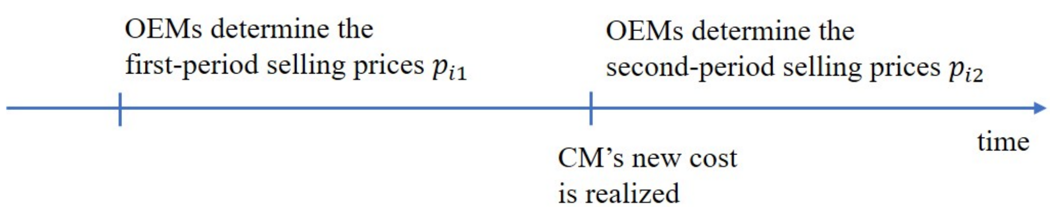


Figure 2.2: Sequence of events in base model

The OEMs are strategic decision makers. Specifically, the OEMs' objective function is the total profit of the two periods, considering the intertemporal influence of learning-by-doing effect. Therefore, the sequence of events is as follows: at the beginning of the first period, the OEMs, knowing the CM's markup and first-period cost, simultaneously determine the retail price to maximize the total profit of the two periods anticipating the cost saving induced by learning effect in the future. At the end of the first period, the first period demands realize and the unit production cost of CM updates. Then, the OEMs simultaneously determine the second-period price to maximize the profit. Moreover, we need the following regularity assumptions to guarantee the meaningful results.

Assumption 1

(a) $\lambda < \frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}$ for any $\theta > 0$,

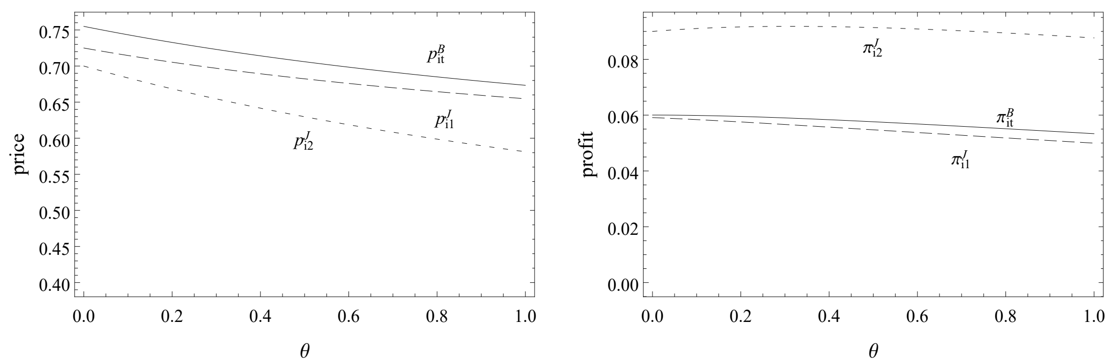
(b) $\frac{4(\theta+1)\lambda^2+2\lambda(\theta+2)^2(\theta+1)}{(\theta+2)^3+2\lambda(\theta+2)^2(\theta+1)}(a-k) < c < a-k$.

Assumption 1 (a) guarantees the positive and finite production quantities of retailers. We can check that $\frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}$ is increasing with θ for $\theta > 0$ and the lower bound for λ is $\sqrt{2}$ when $\theta = 0$. The learning speed can't be too high compared to the competition intensity; otherwise the OEMs would charge negative prices to produce more. Assumption 1 (b) guarantees the positive margins and positive input costs. It is derived from the condition that $c - \lambda(q_{11} + q_{21}) > 0$. Similar assumptions can be found in Keskinocak and Savasaneril (2008) and Chen and Roma (2011). The unit production cost couldn't be higher than $a - k$ which is the maximal gross profit margin for the OEM. The unit production cost couldn't either be too low. $\frac{4(\theta+1)\lambda^2+2(\theta+2)^2((\theta+1)\lambda)}{(\theta+2)^3+2\lambda(\theta+2)^2(\theta+1)}$ is a ratio between 0 and 1 when the condition of Assumption 1 (a) is satisfied. Therefore, it is equivalent to say that the unit production cost should be larger than a proportion of the maximal gross profit margin.

We'll use the superscript J to denote the results under base model. The equilibrium results could be derived in closed-form. All the analyses are left in the appendix. We call the base model joint learning since both OEMs share the benefits of cost reduction from a common CM. Hence, the OEMs are engaged in the so-called coopetition relationship: the competitors are cooperating to enlarge the effect of learning-by-doing by selling more in the first period so that the cost reduction they could benefit in the second period is more significant. But the prospect of cost reduction induces OEMs to price low, possibly intensifying the competition between the OEMs as well. The following proposition compares the prices, quantities, and profits under joint learning and without learning.

Proposition 2.1. *When OEMs outsource to a common CM, OEMs' equilibrium prices* *in both periods are lower and quantities in both periods are higher than the case without learning. OEMs' first-period profit is lower, but the second-period profit and the total profit are higher than the case without learning.*

We find that the joint learning is beneficial for the OEMs and consumers. The OEMs could obtain higher total profit and the customers can enjoy lower prices. Figure 2.3 shows the differences of prices and profits of two periods between the joint learning and the case without learning. The horizontal axis is the competition intensity θ, ranging from 0 to 1. The other parameters are assumed as follows: $a = 1$, $c = 0.5$, $k = 0.01$, and $\lambda = 0.2$. In the case without learning, the prices and profits of both periods are the same, as denoted by p_{it}^B and π_{it}^B in the figure.



(a) Price comparison (b) Profit comparison

Figure 2.3: Price and profit comparisons for joint learning

From subfigure (a), we can observe that the prices in both periods are lower than the price in the case without learning. One thing we would like to emphasize is that the drivers for price reduction are different in two periods. In the first period, OEMs reduce the price for strategic consideration. They price lower to induce larger quantities so that the cost reduction from learning-by-doing in the second period will be more significant. While in the second period, the price was reduced because of the low production cost. Moreover, the price reduction in the first period is always less than the price reduction in the second period. Therefore, the cost reduction effect is stronger than the strategic effect.

When OEMs outsource to a common CM, competition and cooperation get entangled. The lower prices in the first period manifests fiercer competition between the OEMs. On the other hand, larger production quantities in the first period also represent deeper cooperation between the OEMs to lever the cost reduction.

From subfigure (b), we can find that the first-period profit is less than the case without learning but the second-period profit is larger than the case without learning. And in total, joint learning brings positive benefit to the OEMs. The decrease of the first-period profit could be viewed as the expense to leverage more cost reduction in the second period. The OEMs would like to afford the profit loss in the first period for the benefit in the second period.

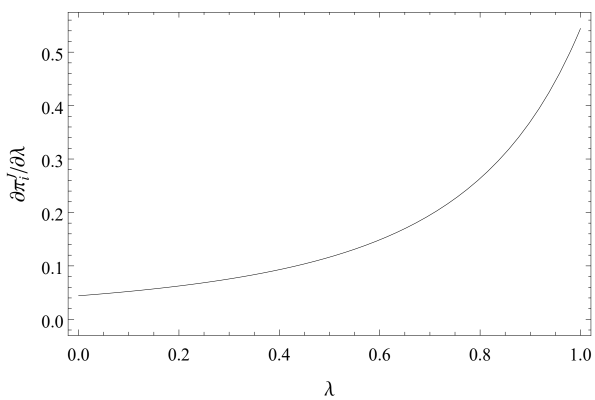


Figure 2.4: Static analysis of π_i^J with λ

We also find that the two-period total profit is increasing with the learning speed λ, as shown in the above figure. The efficiency of learning could be fully exploited when OEMs outsource to a common CM.

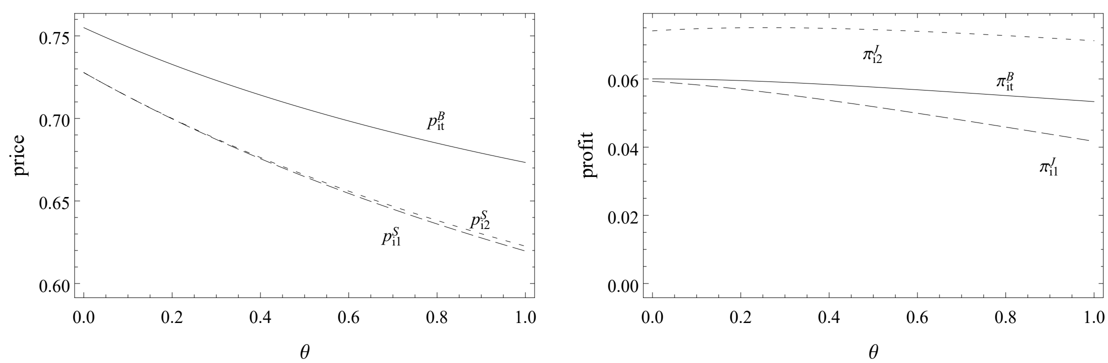
2.3.2 Separate Learning

We now consider the case that OEMs outsource to different CMs. This is also a common practice in reality. We call this scenario "separate learning" to differentiate with the base model. Under separate learning, two OEMs do not have the cooperation relationship. They only compete in the final market. It is a useful auxiliary model for us to isolate the competition effect from the cooperation effect. With separate learning, OEMs competition behavior may be different from the case without learning. This is called the competition effect of learning-by-doing. To simplify the analysis, we assume that the CMs charge the same markup k to the production, which could be viewed as the prevailing price of the industry. Under these circumstances, is the learning-by-doing effect profitable to the OEMs? Will the learning-by-doing effect influence the competition?

The superscript S is used to represent the scenario of separate learning. The events sequence is the same as base model. The only difference is that the cost reduction in the second period for either OEM depends only his own first period production quantity. With the equilibrium results, we can check the influence of separate learning by examining the comparison with the case without learning.

Proposition 2.2. *When OEMs outsource to separate CMs, OEMs' equilibrium prices in both periods are lower and quantities in both periods are higher than the case without learning. OEMs' first-period profit is lower and the second-period profit is higher than without learning. Moreover, when* $0 < \theta < 1.7938$ *and* $0 < \lambda < \lambda_1(\theta)$*, OEMs' total profit with separate learning is higher than without learning, otherwise, the total profit with separate learning is lower than without learning.*

When the OEMs do not cooperate in learning, the learning-by-doing effect only influences the competition. As we can observe in Figure 2.5, the general influence of separate learning to OEMs' prices and profits in either period is similar to joint learning. With separate learning, prices are lower and selling quantities are higher than without learning in both periods. The price of the first period is lower than the price of the second period. Therefore, the strategic effect is stronger than the cost reduction effect, contrary to the joint learning.



(a) Price comparison (b) Profit comparison

Figure 2.5: Price and profit comparison for separate learning

From subfigure (b), we find that the first period profit is lower and the second period profit is higher than the case without learning, similar to joint learning. However, the influence of separate learning on the OEMs' total profit is not consistent with joint learning. When the competition intensity is relatively large or when the learning speed is large enough, the profit increase in the second period is not sufficient to cover the profit loss in the first period. Therefore, separate learning may have negative influence on the OEMs' total profit despite the cost reduction derived from learning-by-doing.

The main reason for the lower total profit under separate learning is the intensified competition. Figure 2.6 depicts the first order derivative of two-period total profit in competition intensity under separate learning and without learning. The total profit of OEMs is decreasing with the competition intensity θ either when OEMs outsource to separate CMs or when there's no learning-by-doing effect. However, the slope is steeper with separate learning than without learning. Therefore, under separate learning, the

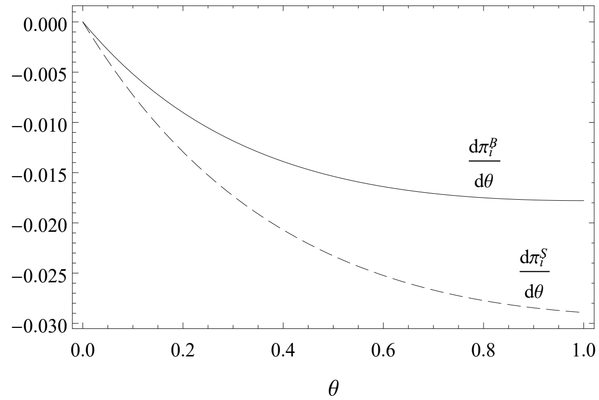


Figure 2.6: Intensified competitions by separate learning

learning-by-doing effect intensifies the competition between two OEMs.

The competition surely provides a new angle for us to understand the learning-by-doing effect. In bilateral monopoly supply chain with one CM and one OEM, the learning effect always brings positive profit gaining to the OEM, appearing as a consistent favorable factor for the OEM. This is easy to understand: without the strategic consideration of competition, the OEM could optimize the pricing decisions to secure the benefit of learning-by-doing without bearing excessive cost. Actually, one stream of literature on learning-by-doing effect incorporates the learning feature for many other centralized operational decisions to figure out more realistic policies, such as Mazzola and McCardle (1997), Fine and Portues (1989), Berstein and Kök (2009).

We also find that when the competition intensity and learning speed are relatively large, OEMs' two-period total profit under separate learning might decrease with λ. The higher learning efficiency can make it easier to achieve cost reduction. This seemingly beneficial characteristics may have negative influence on the total profit of the OEMs under separate learning, due to the intensified competition.

2.3.3 Analysis

Based on the equilibrium results under joint learning and separate learning, we're ready to investigate the behavior of competition effect and cooperation effect. The first result is that the cooperation induced by the joint learning is beneficial to the OEMs.

Lemma 2.3. *OEMs' total profit is higher under joint learning, compared to the case of separate learning.*

As we observed from the former subsection, separate learning intensifies the competition effect and may have negative influence on the OEMs' total profit. Under joint learning, the OEMs cooperate in the sense that the cost reduction achieved is shared by the OEMs. The pooled cost reduction benefits both OEMs and dominates the negative influence of intensified competition. The implication of the above result is that the OEMs would prefer to outsource to a common CM if they are symmetric in the market base. The proposition below shows the static analysis of OEMs' two-period total profit in competition intensity under joint learning.

Proposition 2.4. *Under joint learning, when $0 < \lambda \leq \frac{1}{4}(\sqrt{17}-1)$, the OEMs' two-period total profit increases with θ when $0 < \theta \leq \theta_1(\lambda)$ and decreases with θ when $\theta > \theta_1(\lambda)$; when $\lambda > \frac{1}{4}(\sqrt{17}-1)$, the OEMs' total profit is decreasing with θ for any $\theta > 0$.*

According to our traditional understanding, competing OEMs' profit should be decreasing with the competition intensity. This phenomenon could be identified under either separate learning or no learning scenario. Proposition 2.4, however, reflects an abnormal phenomenon that the manufacturer's profit might be increasing with the competition intensity as shown in Figure 2.7.

To understand the driving forces, it is useful for us to isolate competition effect from cooperation effect. The competition effect of learning-by-doing is the decreasing

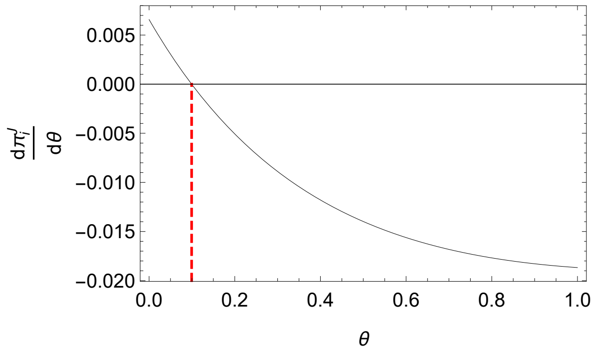
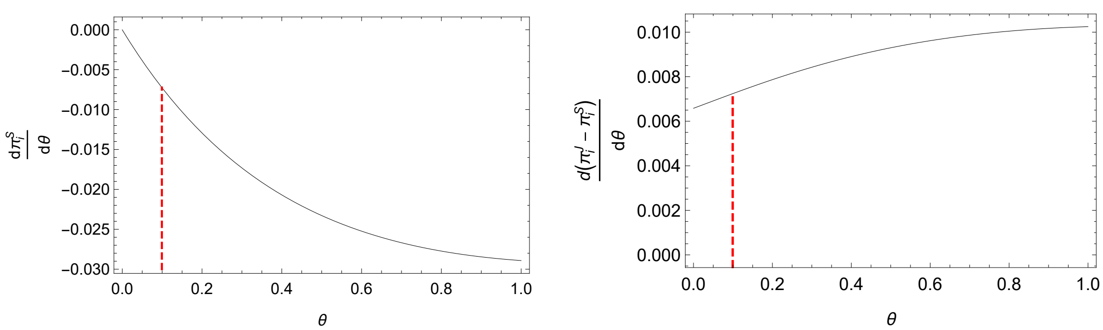


Figure 2.7: Static analysis of π_i^J with θ

trend of the two-period total profit under separate learning with respect to competition intensity, as shown in the following Figure 2.8 (a). The cooperation effect of learning-by-doing is defined as the first order tendency of the profit difference between joint learning and separate learning in competition intensity, which is illustrated in the Figure 2.8 (b) below.



(a) Competition effect

(b) Cooperation effect

Figure 2.8: Analysis of competition and cooperation effect

When λ is less than the threshold $\frac{1}{4}(\sqrt{17}-1)$, the cooperation effect dominates the competition effect when θ is relatively small. Hence, the total profit is increasing with the competition intensity. With the increasing of competition intensity, the competition effect is getting strengthened at a higher speed. Finally, when θ is larger than $\theta_1(\lambda)$, the competition effect dominates the cooperation effect. Increasing competition intensity would mainly lead to fiercer competition, decreasing the OEMs' profit.

The underneath mechanism could be understood from the comparison of prices and profits in either period under joint learning and separate learning. We find that with joint learning the second-period price is always lower and the second-period profit is always higher, compared to separate learning. So, OEMs can always benefit from larger pooled cost reduction under joint learning. Moreover, when λ and θ are relatively small, the first-period price and the first-period profit would also be higher under joint learning. Under this situation, the larger pooled cost reduction was contributed by smaller individual first-period quantity, reflecting the dominated role of cooperation effect. By cooperating in the joint learning, the OEMs bear a smaller expense in the first period to leverage larger cost reduction in the second period.

The implication of the proposition is that the role of competition intensity is two-fold under joint learning. Larger competition intensity represents fiercer competition, aggravating the negative influence of learning-by-doing. On the other hand, competition intensity leads to lower prices and larger quantity for competing OEMs which results in more pooled cost reduction which represents more positive influence of cooperation effect with joint learning.

We also find that the total profit under joint learning is increasing with learning speed λ for any given θ. With the cooperation on learning, the threaten that the magnified cost reduction achieved by high learning efficiency may drag OEMs into excessive competition is neutralized.

2.4 Discussions

In this section, we would discuss the influence of joint learning in different aspects to enrich our understanding. In the first subsection, we'll investigate another pricing scheme in which both OEMs would commit to a intertemporally consistent price. Then, we'll verify whether the OEMs would be better if they are myopic in pricing making. Thirdly, we'll investigate the influence of CM's pricing power. In the last subsection, we'll use a numerical example to show the influence of asymmetric OEMs.

2.4.1 Uniform Pricing

In the base model, we follow the regular pricing behavior of OEMs who charge different prices at two periods. In this subsection, we study the equilibrium when OEMs charge uniform prices in the two periods. This price is a commitment lasting for two periods. We call the pricing scheme in the base model as differential pricing for reference.

When the price has commitment effect, the problem is reduced to one-shot decision. OEMs determine the prices at the beginning of the first period simultaneously. The prices are also executed in the second period, although CM updates the production cost at the beginning of the second period.

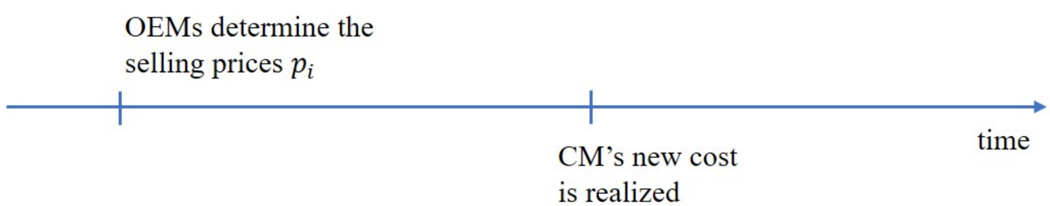


Figure 2.9: Sequence of events under uniform pricing

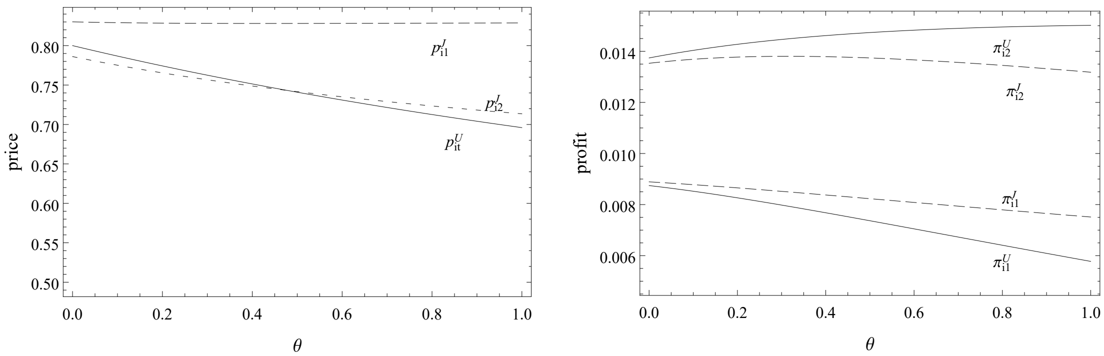
To guarantee the non-negativity of the equilibrium results, we assume that $0 < \lambda < \frac{4+2\theta}{3+2\theta}$ for any $\theta > 0$. The effect of the competition intensity is also two-fold. It intensifies the competition but also facilitate the cooperation. The following proposition states the sensitivity of the total profit in competition intensity.

Proposition 2.5. *With uniform pricing, when $0 < \lambda \leq 1$, the OEMs' two-period total profit increases with θ when $0 < \theta \leq \frac{\lambda}{2-2\lambda}$ and decreases with θ when $\theta > \frac{\lambda}{2-2\lambda}$; when $\lambda > 1$, the OEMs' two-period total profit is decreasing with θ for any $\theta > 0$.*

When the learning speed is relatively small, the cooperation effect firstly dominates and then the competition effect dominates with the increasing of competition intensity. When the learning speed is large, the learning effect is efficient to achieve cost reduction through cooperation. The cost advantage is sufficient to cover the sacrificed profit in the first period. With the increasing of competition intensity, OEMs charge lower price which leads to larger quantity and larger cost reduction. The cooperation effect is playing a dominating role. The two-period total profit is increasing with the deepening cooperation resulted from the increased competition intensity. The following proposition compares the uniform pricing and differential pricing.

Proposition 2.6. *The uniform pricing scheme has the following properties:*
(1) The price of uniform pricing is always lower than the first-period price in the base model.
(2) For any $\theta > 0$*, when* $0 < \lambda < \frac{4+12\theta+9\theta^2+2\theta^3}{4+18\theta+18\theta^2+4\theta^3}$*, the price of uniform pricing is higher than the second-period price in the base model; when* $\lambda > \frac{4+12\theta+9\theta^2+2\theta^3}{4+18\theta+18\theta^2+4\theta^3}$*, the price of uniform pricing is lower than the second-period price in the base model.*
(3) For any $\theta > 0$*, the OEMs would prefer differential pricing when* $0 < \lambda \leq \lambda_2(\theta)$ *and prefer uniform pricing when* $\lambda > \lambda_2(\theta)$*.*

We find that when OEMs commit to an intertemporal consistent price, they would charge a price lower than the first period price of differential pricing. When OEMs price strategically, they can freely charge a low price in the second period to exploit the cost advantage. Under uniform pricing, OEMs have to reduce price to accommodate to the need of low price in the second period. The first-period profit would also be lower and the resulting cost reduction is naturally larger under the uniform pricing. The profit loss in the first period could viewed as the expense to smooth the price. When the learning speed is sufficiently high, the uniform price is even lower than the second-period price in strategic pricing and the achieved low cost is sufficient to make up the expense. Figure 2.10 shows the price and profit comparison between uniform pricing (with superscript U)and differential pricing (base model with superscript J).



(a) Price comparison (b) Profit comparison

Figure 2.10: Price and profit comparisons for uniform pricing

Price commitment prevents the OEMs to strategically utilize the second-period cost reduction. It asks the OEMs to smooth the price instead. This smoothing behavior is beneficial only when the learning effect is sufficiently large. The implication from the proposition is that when the learning speed is relatively small, the differential pricing can prevent OEMs from over-exploiting the learning effect and competing intensively. However, when the learning speed is large, the differential pricing is not sufficient to exploit learning effect.

2.4.2 Myopic Pricing

There're cases when OEMs are myopic in the sense that OEMs maximize the sole first period profit when deciding the first-period prices instead of strategically considering the future cost reduction. How would the OEMs' total profit change? What's the influence of learning-by-doing effect on the OEMs? We'll use the superscript 'M' to differentiate this scenario.

In the first period, the myopic OEMs determine the first-period prices to maximize the sole first-period profit. The first-period decision is just as the case without learning-by-doing effect. At the end of the first period, the equilibrium demand is realized. The production cost is updated as well. Then, at the beginning of the second period, OEMs determines the second-period prices to maximize the second-period profit.

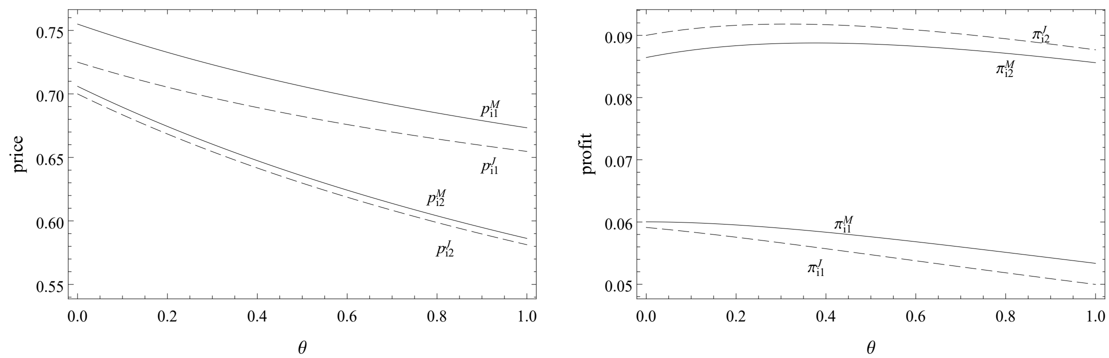
Although the cooperation relationship still exists for myopic OEMs, will the cooperation effect dominate the competition effect under this circumstance? The following proposition states the sensitivity of the total profit in competition intensity.

Proposition 2.7. *When both OEMs are myopic, for any given $\lambda > 0$, the total profit is increasing with θ when $0 < \theta < \theta_2(\lambda)$ and then decreasing with θ when $\theta > \theta_2(\lambda)$.*

We find that for the full range of the learning speed, the competition intensity sways the relative strength of competition effect and cooperation effect. When the competition intensity increases, the price in the first period decreases, enlarging the cost reduction in the second period. The competition effect is dominated by the cooperation effect when the competition intensity is relatively small. The following proposition compares the myopic pricing and differential pricing.

Proposition 2.8. *When both OEMs are myopic, the prices in both periods are always higher than the base model. The first-period profit is always higher and the second period profit is always lower than the base model. However, the two-period total profit is higher than the base model when $0 < \lambda \leq \lambda_3(\theta)$ for any $\theta > 0$.*

The results are depicted in Figure 2.11. Myopic OEMs do not strategically price low in the first period, the first-period profit is at optimal without additional expenses to strategically take advantage the learning-by-doing effect. The cost reduction in the second period is less than the base model. Therefore, the second-period profit of myopic OEMs is less. For OEMs outsourcing to a common CM and strategically considering the future cost reduction, when the learning speed is relatively small, the expense of strategically price in the first period is less than the profit gained in the second period. Therefore, the total profit under myopic pricing is higher.



(a) Price comparison (b) Profit comparison

Figure 2.11: Price and profit comparisons for myopic OEMs

We also find that when myopic OEMs outsource to different CMs, the total profit is always higher than the case without learning. Therefore, when OEMs are myopic, the learning-by-doing effect won't aggravate the competition effect. The total profits are increasing with the learning speed no matter myopic OEMs outsource to a common CM or separate CMs.

2.4.3 CM's Pricing Power

In this subsection, we consider the situation when Contract Manufacturer (CM) has some pricing power in stead of playing an inactive role in the base model. As Gray et al. (2009) say, the CM is playing a more and more important role in the supply chain. A plausible assumption is that CM functions as the Stackelberg-follower in the supply chain. To characterize the decision of the CM, we decompose the retail price in each period as production cost c_t, wholesale margin k_{it} and retail margin as m_{it}. That is, the price in each period could now be written as: $p_{it} = c_t + k_{it} + m_{it}$.

The sequence in each period is as follows: OEMs determine the retail margins first to maximize the two-period total profit. Then CM, given the retail margins, determines the mark-up afterwards to maximize the two-period total profit. The specific decision sequence could be found in the following figure.

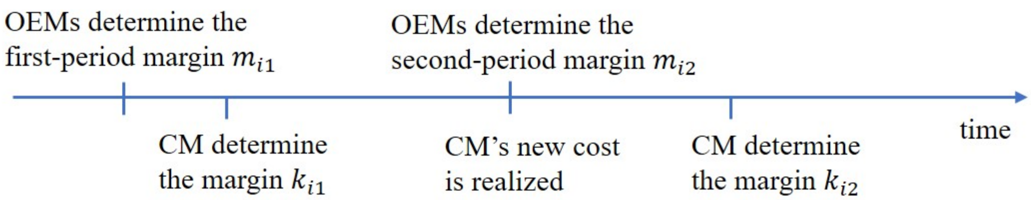
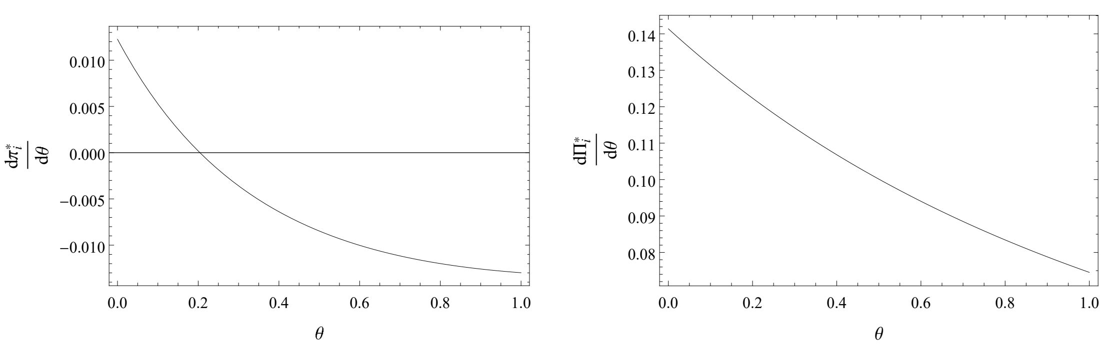


Figure 2.12: Sequence of events when CM has pricing power

The equilibrium results could be derived in closed-form, please refer to the appendix for the derivation of the equilibrium results. When CM has pricing power, he could get a share from the cost reduction. The pooled cost reduction benefits the OEMs less.

From the static analysis with competition intensity, we find that when the CM has pricing power, the OEMs' total profit may firstly increase with θ and then decrease with θ. In the meanwhile, the CM's total profit is increasing with the competition intensity.



(a) The impact of competition intensity on OEMs' profit

(b) The impact of competition intensity on CM's profit

Figure 2.13: The impact of competition intensity when the CM has pricing power

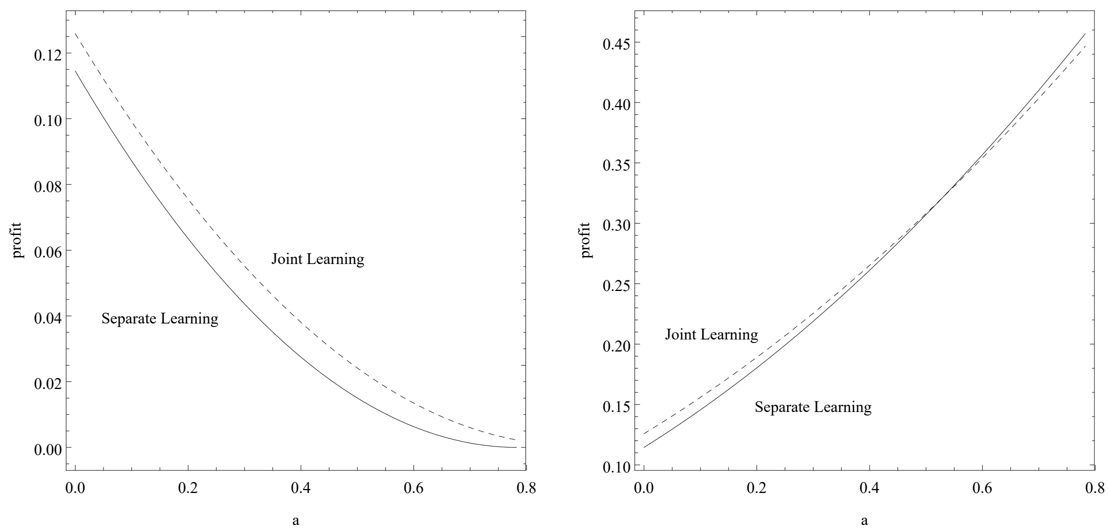
Figure 2.13 (a) shows the first order derivative of the OEMs' total profit with θ. Figure 2.13 (b) shows the first order derive of CM's total profit with θ. The implication from the above result is that the dominating role of cooperation effect is robust when CM has the pricing power. The CM also benefits from the increasing competition intensity. Therefore, the cooperation effect does not depend on the channel power of OEM. As long as OEMs outsource to a common CM, the pooled cost reduction could function as a mechanism facilitating the cooperation between the OEMs.

2.4.4 Asymmetric OEMs

In this subsection, we examine the scenario where OEM have asymmetric market sizes. We have known that OEMs should choose a common CM to take advantage of the joint learning effect when they are symmetric, compared to the separate learning. In reality, the OEMs may be differentiated in the market sizes. Would the asymmetric OEMs have the same preference in terms of the outsourcing choice?

Without lose of generality, the market size of OEM 1 is $1-\alpha$ and OEM 2 is $1+\alpha$, with $0 < \alpha < 1$. From the numerical study in Figure 2.13, we find that OEM 1 always prefer a common CM to leverage the pooled learning-by-doing effect. OEM 2, however, would prefer separate learning when α is relatively large.

The trade-off is as follows: the OEM with a larger market size produces more, contributing more to the cost reduction. The lower cost in the second period, however, enables the OEM with a smaller market size to compete more aggressively. The common CM gives the OEM with a smaller market to seize a free ride to take advantage of the cost reduction. Therefore, the coopetition relationship may not be favorable for OEM with a larger market. When the OEM's market size is large enough, he would not like to outsource to a common CM.



(a) OEM's profit with market size $1 - \alpha$ (b) OEM's profit with market size $1 + \alpha$

Figure 2.14: Profits comparisons for asymmetric OEMs

2.5 Conclusion

This paper studies the coopetition effect of learning-by-doing in a two-period model. We consider the problem in a supply chain where two competing OEMs outsourcing to a common CM for production. CM's second-period production cost decreases linearly with the total production quantities produced in the first period. The OEMs cooperate in enhancing the CM's learning for more cost reduction while competing through Bertrand model in either period. With the strategic considering of the cost reduction, OEMs in the first period will determine the price to maximize the total profit of the two periods. We focus on the strategic interaction of the OEMs and assume that the CM plays a passive role in the base model.

To disentangle the problem, we isolate the competition effect from cooperation effect by analyzing an auxiliary model where the OEMs outsource to separate CMs. We find that the learning-by-doing effect intensifies the competition between the OEMs. With the increasing of competition intensity, the two-period total profit under separate learning is decreasing at a quicker speed than the case without learning. Therefore, the learning-by-doing effect may have a negative influence on the OEMs' total profit. This shows a salient feature of learning-by-doing effect in competition. Characterizing the learning effect, decision-makers should incorporate the learning effect to optimize the operational strategies.

When OEMs outsource to a common CM, the pooled cost reduction functions as complementary resource for the OEMs, making learning-by-doing always preferable for the competing OEMs. The role of competition intensity is two-fold. The increasing of competition intensity manifests fiercer competition but also facilitates the cooperation since the OEMs price lower in the first period and the effect of learning is more significant. When the competition intensity and the learning speed are relatively low, the total profit of OEM is increasing with the competition intensity, reflecting the dominant role of cooperation effect. The dominant cooperation effect is robust when considering the pricing power of CM and other pricing strategies including uniform pricing and myopic pricing.

The managerial implication is that the manufacturers should be cautious to strategically leverage the learning-by-doing effect when the competitor outsources to separate CMs while the pooled cost reduction when outsourcing to a common CM always makes the learning-by-doing effect profitable. Playstation 4 was introduced with tight margin, with the estimated production cost \$381 and the initial selling price \$399. The low introduction price brings large amount of sales for Sony since Playstation 4 was put into the market. The cost reduction enables Sony to cut price after a period of the introduction. Playstation 4 is about to be the product with the highest sales among all game consoles, demonstrating huge success of this product. Despite the marketing momentum, manufacturing efficiency accounts a lot. The increasing of competition intensity might be a good thing for OEMs who outsource to a common CM.

We also find that although joint learning is more preferable for symmetric OEMs, the strategy of outsourcing to a common CM is not optimal for OEMs with different market sizes. The OEM with a significantly larger market size may prefer to outsource to separate CM to avoid the competitor's free ride on cost reduction. One possible direction for future research is to extend the problem into multiple periods. What's the optimal policy of the OEMs? In the multi-period setting, will the discount rate influence the results?

Chapter 3

Trade Credit with Supply Chain Competition

3.1 Introduction

Trade credit is the most common contract through which the supplier provides financial support for the downstream retailers either in the developed economies or less developed countries (Fisman and Love 2003). It is the single largest source of short-term firm financing (Petersen and Rajan 1997). It was estimated that trade credit funded almost 90% of global merchandise trade in 2007, amount to 25 trillion dollars (Klapper et al. 2012). In China, the trade credit amounts to 9.1% of the firm's total asset on average, observing from 674 firms listed in the Shanghai and Shenzhen Stock Exchanges (Cai et al. 2015). Compared with the professional financial intermediates, suppliers are endowed with advantage to act the financial function in supply chain. For example, supplies may have more information about the demand or have privilege to prevent retailer diverting cash.

Researchers in operations have recently started to investigate the effect of trade credit in the supply chain structure. Trade credit could improve the supply chain efficiency with the risk-sharing effect (Yang and Birge 2017). In the vertical one-supplier-one-retailer supply chain, the retailer prefers supplier finance to the competitive bank finance and supplier is always willing to provide trade credit at a rate lower than the risk-free interest rate (Kouvelis and Zhao 2012). Moreover, trade credit can affect the traditional supply chain contract in coordination (Xiao et al. 2016). In the horizontal competition, trade credit also changes the firm's competitive behavior. Yang et al. (2017) proves that offering trade credit, firms in Bertrand competition would raise the price above cost and thereby soften the competition.

Suppliers are not satisfied with limited commercial map. The supply channel expands with the growth of business. With the spreading scale of supply channel, downstream retailers are differentiated with the financial statuses. Moreover, the downstream retailers are usually from regions with different bank-firm relationships. In developing economy like China, the development of commercial finance intermediates is insufficient and severely unbalanced in different regions. Yano and Shiraishi (2014) employ data to study the factors that influence the development of trade credit among different regions in China. The situation for suppliers who have business all over the world is more salient. In reality, suppliers usually offer retailers trade credit with different terms to expand the sales. For example, Ocean Trawler is a company which supplies customers with various fish, including halibut, Alaska Pollock and herring. The business is operating worldwide including both developed and less developed regions. Trade credit is a common strategy for Ocean Trawler to expand sales in traditional markets like UK and Dutch. Offering more flexible trade terms, Ocean Trawler is able to develop business in new markets like Togo as well. Although the effect of trade credit in the isolated vertical or horizontal relationship of supply chain is well understood, what's the role of trade credit in supply chain with downstream competition is not clear.

To the best of our knowledge, we are the first to investigate the trade credit in supply chain with downstream competition. This study aims to investigate the effect of trade credit on the interaction of vertical and horizontal relationship. Specifically, (1) what's the effect on the supply chain decisions such as wholesale price and selling quantity? (2) What's the optimal specification of trade credit contract terms? (3) What's the strategic implication of trade credit to the supply chain members?

Our research features a supply chain consisting of one supplier and two competing retailers. In particular, the supplier produces and supplies two retailers who are selling partial substitutes in competing markets. The exact of the market size is unknown when retailers determine the order quantities. The demand shock brings risk to the future cash flow of the supply chain and integrates the finance and inventory decisions endogenously. The distribution of the demand shock is assumed as common knowledge among the supply chain parties.

The financial status of the supply chain members are different. The downstream retailers are mostly small-to-medium sized firms who might be faced with financial constraint. The external financial resource for the small-size firm is usually limited. To facilitate the analysis, we assume extreme financial statuses of retailers: with no initial capital or sufficient capital. The supplier, on the other hand, is large-scale manufacturer with a strong financial status and sufficient financial resources. Therefore, the supplier offers trade credit at an interest rate stipulated by the contract if the retailer is lack of initial capital. Retailers repay the trade credit after they seize the revenue at the end of selling season. If the operations revenue is insufficient for the repayment, the retailer files for bankruptcy and the revenue is liquidated by the supplier.

We analyze the problem as a Stackelberg game between the supplier and the retailers, with the supplier acting as the leader and retailers as the follower. The supplier determines the wholesale price and trade credit interest rate at the beginning of the selling season. Retailers then determine the selling quantities in accordance with their financial statuses. After the realization of market condition, retailers seize the revenue and repay the trade credit accordingly.

Our results are summarized as follows. The equilibrium results are segmented into two parts: no bankruptcy risk range and bankruptcy risk range. In the range of no bankruptcy risk, the problem is reduced to the benchmark case where both retailers have sufficient capital. Increasing the variance of demand shock and downstream competition intensity can drive the financially distressed retailer into the bankruptcy risk range.

In the presence of downstream competition, the supplier obtains a higher expected profit by offering trade credit. This is consistent with the prevalence of trade credit practices and the theoretical explanations for the existence of trade credit. Offering trade credit gives the supplier a chance to charge a higher wholesale price to compensate for assuming the market risk. When the retailers have unbalanced financial statuses (i.e., one retailer has no initial capital while the other has sufficient capital), the supplier offers a lower adjusted wholesale price compared to the case where both retailers are financially distressed. This is parallel to the bail-out effect mentioned in Yang et al. (2015). However, the wholesale price for the retailer with sufficient capital is not influenced by the bankruptcy risk of the competitor. Therefore, we do not observe the abetment effect mentioned in Yang et al. (2015). Moreover, the supplier may prefer the case of unbalanced financial statuses of the downstream retailers when the variance of demand shock is in an intermediate range. For a given variance of demand shock, the supplier may first prefer the unbalanced financial status and then the case where both retailers are financially distressed with the increasing of competition intensity.

When both retailers are financially distressed, the trade credit could aggravate the downstream competition when the variance of demand shock is relatively large. When the retailers have unbalanced financial status, we observe bidirectional predatory behavior. When the variance of demand shock is moderate, the selling quantity from retailer with sufficient capital is higher, predating the retailer with no initial capital. When the variance of demand shock is large, the retailer with no initial capital would sell more than the retailer with sufficient capital. The profitability of the retailer with sufficient capital is always higher no matter what the financial status of the competitor is. On the other hand, the change of competitor's financial status has more subtle effect. Given the intensity of competition, when the variance of demand shock is moderate, competitor's enhancement in financial status cuts the retailer's profit; when the variance of demand shock is high, retailer's profit will increase with the improvement of competitor's financial status.

The rest of this chapter is organized as follows. We summarize the related literature in section 3.2. We formulate the model in section 3.3. The equilibrium results and related analysis are presented in section 3.4. We conclude the result and present future research directions in section 3.5.

3.2 Literature

Many researches explain the coexistence of trade credit with bank credit by investigating the relative merits of trade credit. Earlier findings mainly study issues from the perspective of finance and economic, including price discrimination (Brennan et al. 1988), lower transaction costs (Emery 1984), tax savings (Brick and Fung 1984), etc. Standing at the upstream, the supplier has advantage in assessing buyers' creditworthiness, salvaging the collateral (Mian and Smith 1992), preventing the opportunistic diversion (Burkart and Ellingsen 2004, Chod 2016). Please refer to Petersen and Rajan (1997) for a comprehensive review on trade credit in finance. The relationship between trade credit and bank credit is unclear. Most evidences support that trade credit functions as substitutes for bank credit (Atanasova and Wilson 2003, Ge and Qiu 2007, Mateut et al. 2006). However, other evidences suggest that trade credit is a complementary to the bank credit (Biais and Gollier 1997, Cook 1999, Babich et al. 2012).

Trade credit also shows operational effects such as signaling product quality (Lee and Stowe 1993, Long et al. 1993), deterring the supplier's moral hazard (Kim and Shin 2012, Babich and Tang 2012, Rui and Lai 2015). The presence of trade credit also influences the firms' inventory strategy (Haley and Higgins 1973, Gupta and Wang 2009, Luo and Shang 2014). However, we are more related to the researches that investigate the influence of trade credit on the interaction of the supply chain members' operational decisions. Kouvelis and Zhao (2012) show that in the one-to-one vertical supply chain relationship, supplier is willing to offer trade credit at the interest rate lower than that the bank would like to charge. The supply chain efficiency improves and the supplier results in higher profit. Based on a similar selling-to-newsvendor setting, Yang and Birge (2017) demonstrate the (demand) risk-sharing role of trade credit contract in vertical supply chain relationship. The optimal trade credit contract term is contingent on the retailer's financial and operational factors, i.e. the retailer's initial capital and market power and supplier's relative efficiency of collecting default claims. The retailer's optimal finance strategy is a portfolio of trade credit and bank loan. Xiao et al. (2016) show that the supply chain coordination is changed by trade credit. Lee and Rhee (2011) show that trade credit could be used as a tool by supplier to achieve supply chain coordination. In contrast with this line of research, our research investigate the usage of trade credit in the supply chain with competing retailers.

In addition to the effect in vertical relationship, Peura et al. (2017) focus on the horizontal benefit of trade credit. Casting on Bertrand competition, the result shows that trade credit softens horizontal price competition in the sense that the equilibrium price is higher than the marginal cost. However, the empirical evidence is not consistent. McMillan and Woodruff (1999) show that competition may reduce the use of trade credit while Hyndman and Serio (2010) find that monopolies are less likely to offer trade credit than suppliers in competition. To the best of our knowledge, we are the first to study how the downstream competition might influence the usage and effectiveness of trade credit.

Our paper is also related to the research on the interaction of product market competition and debt raising issue. Brander and Lewis (1986) is the seminal paper investigate the impact of firms' capital structure on the product market behavior. Showalter (1995) extends the model to the Bertrand competition. Wanzenried (2003) shows the relationship between firm's output market and capital structure decisions with respect to specific demand and supply characteristics. Specifically, with higher demand volatility or lower substitutability, the equilibrium debt level is higher. However, these financial research consider the debt from the pure strategic perspective. We link the amount of trade credit to the production quantities ordered in competitive market.

Another concern about trade credit is the determinants of contract terms. The finance researches now mainly agree on the net present value approach to calculate the optimal payment terms (Kim and Feist 1995). However, the formula requires estimates in how changes of credit terms impact demand. Operations researchers can apply insights from inventory control to predict these changes (e.g., Schiff and Lieber 1974, Abad and Jaggi 2003, Shi and Zhang 2010, Kouvelis and Zhao 2011). In our model, supplier determines the interest rates of trade credit considering the financial statuses of the retailers, the competition intensity and the market uncertainty.

Finally, our research also relates to the broader operations-finance interface which investigates the interaction between the firms' operational decisions and financial problems. This rapidly growing area includes Babich and Sobel (2004), Buzacott and Zhang (2004), Xu and Birge (2004), Dada and Hu (2008), Lai et al. (2009), Boyabatlı and Toktay (2011), Li et al. (2013) and Yang et al. (2015). In fact, the characteristics of trade credit naturally involves the joint consideration from both operations and finance areas. Seifert et al. (2013) provide an integrated multi-discipline review on the trade credit.

3.3 Model Formulation

To study the usage and effectiveness of trade credit in the joint vertical and horizontal supply chain relationship, we consider a two-echelon supply chain which consists of one supplier selling through two retailers who are competing in uncertain consumer market. The one-supplier-two-retailer structure is commonly used in the operations management literature, see Ingene and Parry (1995), Padmanabhan and Png (1997), Yang et al. (2015), ect. We use subscript $i \in \{s, 1, 2\}$ to represent the supplier and two retailers respectively.

Supplier produces and sells to the retailers at wholesale price w. The production cost of supplier is normalized to zero. The retailers incur no extra selling cost. Retailers are involved in quantity competition with uncertain market condition. The inverse demand function is given by: $p_i = A - q_i - \gamma q_{3-i} + z_i$, $i = 1, 2$. Here, A is the base market size. The retailers are faced with symmetric base markets. The parameter $\gamma \in [0, 1]$ measures the competition intensity between the retailers: the larger γ, the more intensified competition. When $\gamma = 0$, the demands of two retailers are independent and both retailers function as monopolies in separate markets. When $\gamma = 1$, the demands of two products are completely substitutable. Other than the last term z_i, the deterministic part of the inverse demand function is derived from consumers with quadratic utility (e.g., Singh and Vives 1984) and is widely observed in literature (e.g., Cachon and Harker 2002, Feng and Lu 2012). z_i represents an exogenous firm-specific shock which is independently drawn from the uniform distribution with support $(\underline{z}, \bar{z})$, where $\bar{z} > 0$ and $\underline{z} < 0$. The range of the random variable z_i is symmetric with respect to zero, i.e. $\underline{z} = -\bar{z}$. The density function of z_i is $f(z_i) = \frac{1}{2\bar{z}}$ if $\underline{z} < z_i < \bar{z}$ and $f(z_i) = 0$ otherwise. Thus, z_i has zero mean and the variance is $\bar{z}^2/3$. Hence, $\bar{z}$ characterizes the extent of the demand shock. z_i is realized after the retailers place orders to the supplier. However, the distribution of the demand shock is a common knowledge among supplier and retailers. To avoid the unrealistic situation of negative demand, we assume $\bar{z} < A$.

The downstream retailers are mostly small-to-medium-sized firms who might be faced with financial constraint. For small-to-medium-sized firms, seeking financing source is a long-existing problem. The supplier, on the other hand, is large-scale manufacturer with a strong financial status and sufficient financial resources. Trade credit is a commonly applied remedy by which the supplier supports the retailers' financial shortage. Supplier has the inclination to offer trade credit for the retailers. As shown in the literature, the supplier gets lots of advantages to assume the responsibility of creditor. In fact, trade credit is an important short-term financing source. In reality, strong suppliers would like to support the financially distressed retailers with trade credit. The examples are abundant such as GM, P&G and Unilever (Xiao et al. 2016). Companies operating worldwide business, e.g., Ocean Trawler, also would like to provide retailers with trade credit to expand the sale. The financial flow is in reverse direction with the physical inventory flow. The market uncertainty makes the future cash flow risky and integrates the finance and inventory decisions endogenously.

We focus on the dichotomy of retailers' financial statuses: sufficient initial capital

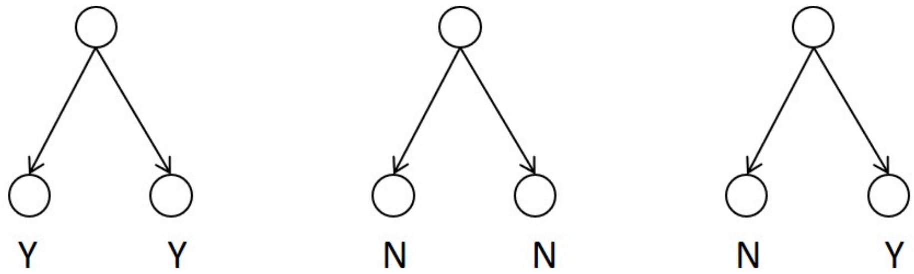


Figure 3.1: Scenarios of retailers' financial statuses

and no initial capital. We use Y to represent the case where the retailer has sufficient initial capital while N represents the case where the retailer has no initial capital. Three possible variants of retailers' financial status combinations are (Y, Y), (N, N) and (N, Y) ((Y, N) is symmetric with (N, Y) and therefore omitted). The first character in the bracket represents the financial status of retailer 1 and the second character represents the financial status of retailer 2 respectively. We treat (Y,Y), where both retailers can be financed with internal capital, as the benchmark. We use the superscript $j \in \{YY, NN, NY\}$ to represent the respective scenarios. For example, q_2^{NY} represents the selling quantity of the retailer who has sufficient initial capital in the scenario of (N, Y).

In reality, trade credit exists in various forms. While in this study, we focus on the following format. At the beginning of the selling season, the retailer anticipates the market condition and determines the order quantity. If he is short of initial capital, he resorts to the supplier for financial support. The trade credit contract stipulates interest rate r_i and repayment date (which is assumed to be the end of the selling season). Let D_i denote the amount of debt which the supplier lends to the retailers. Since we assume that the financially constrained retailer has no initial capital, the debt amount is given by $D_i = w_i q_i$. When the market condition is bad and the revenue of retailer is insufficient to fulfill the required repayment $w_i q_i(1 + r_i)$, the retailer files for bankruptcy and the supplier is compensated with the retailer's total revenue. We do not consider the cost of financial distress. The risk-free interest rate is normalized to zero.

The supplier functions as the Stackelberg leader and the retailers as the followers. The sequence of events is as follows. At the first stage, the supplier determines the wholesale price w_i and the trade credit interest rate r_i. Then, the retailers simultaneously determine the selling quantities q_i to maximize their expected profits. The gap between the required procurement cost and the retailer's capital is filled by the trade credit offered by the supplier. After the selling season, retailer collects revenue and repays the loan if needed. When the market condition is bad, retailer files for bankruptcy and all revenue goes to the supplier.

3.4 Analysis

In this section, we analyze the equilibrium of the supply chain under three possible combinations of the retailers' financial statuses. This helps us understand how would the downstream financial status, competition intensity and market uncertainty influence the contract terms of trade credit. In the first subsection we'll analyze of the benchmark case where both retailers have sufficient capital to support their procurement. Then, we'll proceed to study the cases in which either retailer or both retailers need trade credit to support procurement. Since the retailers are symmetric in the cases of (Y, Y) and (N, N), we only focus on the symmetrical equilibrium. We replace w_i, r_i with w and r in the two scenarios.

3.4.1 Benchmark (Y, Y)

When retailers are endowed with sufficient capital, they determine selling quantities without the threat of bankruptcy liquidation. We derive the equilibrium by backward induction. Given the wholesale price w, the retailers determine the selling quantities to maximize the expected profit.

$$\pi_i^{YY}(q_i) = \int_{\underline{z}}^{\bar{z}} (q_i(A - q_i - \gamma q_{3-i} + z_i) - wq_i) f(z_i) dz_i, \ i = \{1, 2\}$$

Since the distribution of z_i is uniform and symmetric with respect to zero, the mean profit under shock is not biased from the expectation, $\pi_i^{YY}(q_i) = (A - q_i - \gamma q_{3-i} - w)q_i$. The best response function of either retailer is given by the first order condition $q_i^{YY}(w) = \frac{1}{2}(A - \gamma q_{3-i} - w)$. The intersection of retailers' best response functions gives the second-stage Nash equilibrium selling quantities $q_i^{YY} = \frac{A-w}{2+\gamma}$. The equilibrium profit of the retailers, contingent on the wholesale prices, are given by $\frac{(A-w)^2}{(2+\gamma)^2}$.

When retailers have sufficient capital, the transactions are realized simultaneously with the transfer of physical inventory. Therefore, the supplier bears no uncertainty under this circumstance. Expecting the equilibrium selling quantities, supplier determines wholesale prices to maximize the profit, $\pi_s^{YY} = 2wq_i$. The equilibrium wholesale prices would be derived by the first order conditions as well. We summarize the equilibrium results in the following lemma.

Lemma 3.1. *When retailers have sufficient capital, the equilibrium wholesale prices are given by $w^{YY} = \frac{A}{2}$ and the two retailers set the selling quantities at $q_i^{YY} = \frac{A}{2(2+\gamma)}$. The retailer's profit is $\pi_i^{YY} = \frac{A^2}{4(2+\gamma)^2}$ and supplier's profit is $\pi_s^{YY} = \frac{A^2}{2(2+\gamma)}$.*

When the retailers have sufficient capital and the transaction is based on cash delivery, the demand uncertainty is born by the retailers only. Furthermore, the symmetric random shock won't distort the selling quantities from the case of deterministic market demand. From the above result, we can see that the selling quantity is monotonically decreasing with the the intensity of downstream competition γ over the full range $[0, 1]$ while the wholesale price is independent of the competition intensity. The intensity of competition also drives down the profits of retailers and supplier but the decreasing trend of the retailer's profit is more significant.

3.4.2 Equilibrium of (N, N)

In this subsection, we study the case where neither retailer has initial capital and they rely on trade credit. Trade credit appears as accounts payable on the retailers' balance sheet. When the market condition is bad, the retailer's revenue is zero and the supplier collects all the revenue for compensation.

Specifically, a higher realization of market demand z_i generates a higher revenue. For either retailer, there exists a critical shock level $\hat{z}_i \in (\underline{z}, \bar{z})$, which is defined by the following break-even condition: $(A - q_i - \gamma q_{3-i} + \hat{z}_i)q_i = wq_i(1+r)$, $i = 1, 2$, where the market revenue, represented by the left-hand side term, equals the required repayment of trade credit contract terms. If the realized market condition is above the threshold, retailer could seize the profit after paying back the trade credit, otherwise retailers will end up with nothing.

According to the definition, the critical threshold $\hat{z}_i$ depends on the selling quantities of the retailers. Given the competitor's selling quantity, wholesale price and trade credit interest rate, the critical threshold is given by $\hat{z}_i(q_i) = w(1+r_i) + q_i + \gamma q_{3-i} - A$. When $q_i < A - \bar{z} - \gamma q_{3-i} - w(1 + r)$, the critical threshold is less than the lower bound of the random variable support, i.e. $\hat{z}_i < \underline{z}$. That is, retailer's order quantity is low enough to exclude the possibility of default. Therefore, the expected profit of the retailer is given by the following piecewise function:

$$\pi_i^{NN}(q_i) = \begin{cases} \int_{\hat{z}_i}^{\bar{z}}((A - q_i - \gamma q_{3-i} + z_i)q_i - wq_i(1+r))f(z_i)dz_i \\ \qquad\qquad\qquad\qquad \text{if } q_i \geq A - \bar{z} - \gamma q_{3-i} - w(1+r) \\ (A - q_i - \gamma q_{3-i})q_i - wq_i(1+r) \\ \qquad\qquad\qquad\qquad \text{if } q_i < A - \bar{z} - \gamma q_{3-i} - w(1+r) \end{cases}$$

The first segment of the profit function integrates the revenue of retailer from the critical threshold to the upper bound of the distribution. Integrating over the full rang of z_i, the second segment of the profit function is the same as the benchmark. Given the trade credit contract terms and the competitor's decision, the retailer could choose to order conservatively to avoid the possible bankruptcy. Hence, we call the first segment as bankruptcy risk range and the second segment as no bankruptcy risk range.

We derive the equilibrium by backward induction. At the second stage, before learning the market condition, the retailers simultaneously determine the selling quantities given the wholesale price and trade credit interest rate. The first-order condition gives the response function of retailer as follows.

$$q_i^{NN}(q_{3-i}) = \begin{cases} \frac{1}{3}(A - \gamma q_{3-i} - w(1+r) + \bar{z}) & \text{if } q_{3-i} \geq \frac{A - w(1+r) - 2\bar{z}}{\gamma} \\ \frac{1}{2}(A - \gamma q_{3-i} - w(1+r)) & \text{if } q_{3-i} < \frac{A - w(1+r) - 2\bar{z}}{\gamma} \end{cases}$$

When the competitor's order quantity is large, the retailer's profit falls into the bankruptcy risk range. Compared with the no bankruptcy risk range, the variance of demand shock influences the selling quantity in the response function of the bankruptcy risk range. If the retailer files for bankruptcy, he'll get zero revenue. From the perspective of the retailer, it is equivalent that the demand distribution is now truncated from the the critical threshold and hence biased upward. In benchmark (Y, Y), the mean of demand shock is zero and the market demand is independent of the variance. Now, a larger variance of demand shock leads to a higher demand expectation. Hence, the response function in bankruptcy risk range increases with the variance of demand shock. The second-stage equilibrium selling quantities could be obtained by the intersection of the response functions of retailers.

$$q_i^{NN}(w,r) = \begin{cases} \frac{A-w(1+r)+\bar{z}}{3+\gamma} & \text{if } \bar{z} \geq \frac{A-w(1+r)}{2+\gamma} \\ \frac{A-w(1+r)}{2+\gamma} & \text{if } \bar{z} < \frac{A-w(1+r)}{2+\gamma} \end{cases}$$

Given the wholesale price and trade credit interest rate, the retailer's profit is given by

$$\pi_i^{NN}(w,r) = \begin{cases} \frac{(A-w(1+r)+\bar{z})^3}{\bar{z}(3+\gamma)^3} & \text{if } \bar{z} \geq \frac{A-w(1+r)}{2+\gamma} \\ \frac{(A-w(1+r))^2}{(2+\gamma)^2} & \text{if } \bar{z} < \frac{A-w(1+r)}{2+\gamma} \end{cases}$$

The resulting threshold is also represented as the function of w and r, $\hat{z}(w,r) = \frac{(1+\gamma)\bar{z}-2(A-w(1+r))}{3+\gamma}$. The critical threshold in the lower limit of integration generates the cubic term in retailer's profit.

Given a certain $\bar{z}$, increasing the wholesale price and trade credit interest rate may trigger the jump from no bankruptcy risk range to the bankruptcy risk range. This gives us a hint about the supplier's decision of wholesale price and trade credit interest rate in the first stage. Anticipating retailers' best response functions, the supplier determines the wholesale price and trade credit interest rate to maximize her total expected profit. Receiving symmetric revenue from either retailer, the supplier's profit now is related to the demand uncertainty through trade credit:

$$\pi_s^{NN}(w,r) = \begin{cases} 2\left(\int_{\underline{z}}^{\hat{z}}(A-(1+\gamma)q_i+z)q_i f(z)dz + \int_{\hat{z}}^{\bar{z}} wq_i(1+r)f(z)dz\right) & \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } w(1+r) \geq A-(2+\gamma)\bar{z} \\ 2wq_i(1+r) & \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if } w(1+r) < A-(2+\gamma)\bar{z} \end{cases}$$

Parallel to the retailer's profit function, the supplier's profit function is also segmented into two pieces. With higher wholesale price and trade credit interest rate, the profit function falls into bankruptcy risk range. Otherwise, the supplier is guaranteed to collect all the trade credit. Solving the problem, we can derive the wholesale price and trade credit interest rate in equilibrium. The following proposition summarizes the equilibrium results.

Proposition 3.2. *When both retailers have no initial capital and rely on trade credit, the equilibrium wholesale price and the trade credit contract are given by*

$$w^{NN}(1+r^{NN}) = \begin{cases} \frac{1}{3}\left(3A+(\gamma^2+4\gamma+6)\bar{z}-(3+\gamma)\Delta_1\right) & \text{if } \bar{z} \geq \hat{\gamma}A \\ \frac{A}{2} & \text{if } \bar{z} < \hat{\gamma}A \end{cases}$$

and the equilibrium selling quantity of the retailer is

$$q_i^{NN} = \begin{cases} \frac{\Delta_1-(1+\gamma)\bar{z}}{3} & \text{if } \bar{z} \geq \hat{\gamma}A \\ \frac{A}{4+2\gamma} & \text{if } \bar{z} < \hat{\gamma}A \end{cases}$$

The profit of supplier is given by

$$\pi_s^{NN} = \begin{cases} \frac{2}{27}(\Delta_1 - (1+\gamma)\bar{z})\left(6A + (1+\gamma)^2\bar{z} - (1+\gamma)\Delta_1\right) & \text{if } \bar{z} \geq \hat{\gamma}A \\ \frac{A^2}{2(2+\gamma)} & \text{if } \bar{z} < \hat{\gamma}A \end{cases}$$

and the profit of retailer is

$$\pi_i^{NN} = \begin{cases} \frac{(\Delta_1-(1+\gamma)\bar{z})^3}{27\bar{z}} & \text{if } \bar{z} \geq \hat{\gamma}A \\ \frac{A^2}{4(2+\gamma)^2} & \text{if } \bar{z} < \hat{\gamma}A \end{cases}$$

where $\Delta_1 = \sqrt{3A\bar{z} + (1+\gamma)^2\bar{z}^2}$ *and* $\hat{\gamma} = \frac{1}{4(1+\gamma)^2}\left((5+\gamma)\sqrt{\frac{5+\gamma}{2+\gamma}} - 7 + \gamma\right)$.

Since the wholesale price and trade credit interest rate are always combined and appear as $w(1+r)$, we call this term integratedly as adjusted wholesale price. Based on the variance of demand shock, the supplier has two arrangements of trade credit. The retailers, accepting the trade credit, would also compete differently in the second-stage competition. At the threshold point, the supplier obtains equal profits under two sets of trade credit contract term.

Recalling that the possible range of $\bar{z}$ is $0 < \bar{z} < A$, the critical value is also represented as a proportion of the base market size A. The critical ratio $\hat{\gamma}$ defining the bankruptcy risk range is a function of γ, with $\hat{\gamma} = \frac{1}{4(1+\gamma)^2}\left((5+\gamma)\sqrt{\frac{5+\gamma}{2+\gamma}} - 7 + \gamma\right)$, which is decreasing with γ. Increasing both the variance of demand shock and the intensity of downstream competition may force the supply chain into bankruptcy risk range.

When the variance of demand shock is low $\bar{z} < \hat{\gamma}A$, the equilibrium results including the adjusted wholesale price, selling quantity and the profits are the same as benchmark when both retailers have sufficient initial capital. Although the supplier supports retailers with trade credit, the adjusted wholesale price is still $A/2$, independent of the market variance and the intensity of competition.

When the variance of demand shock is large $\bar{z} > \hat{\gamma}A$, the supply chain falls in the bankruptcy risk range. With the link of trade credit, the demand uncertainty transmits risk through the supply chain parties. Since the trade credit exists as the debt liability for the retailers, the possible bankruptcy liquidation influences the retailers' order decisions accordingly.

Corollary 3.3. *In the bankruptcy risk range of (N, N) equilibrium, i.e.,* $\bar{z} \geq \hat{\gamma}A$*, the selling quantities of the retailer have the following relationship: when* $\hat{\gamma}A \leq \bar{z} < \frac{3A}{4(\gamma+2)}$*,* $q_i^{NN} < q_i^{YY}$*; when* $\bar{z} \geq \frac{3A}{4(\gamma+2)}$*,* $q_i^{NN} \geq q_i^{YY}$*.*

There exists a threshold $\frac{3A}{4(\gamma+2)}$, when the variance of demand shock is larger than the threshold, the selling quantities of retailers are larger than the benchmark. The retailers compete more aggressively. The supplier would set the adjusted wholesale price larger than the wholesale price in no bankruptcy risk range to cope with the risk. That is, the supplier is charging a risk premium. The rationale is as follows. The break-even condition of the retailer, $(A - q_i - \gamma q_{3-i} + \hat{z}_i)q_i = w_i(1 + r_i)q_i$, could be understood as the condition that the realized retail price equals to the adjusted wholesale price. Setting the adjusted wholesale price is equivalent to setting the bankruptcy threshold. Therefore, the benefit to increase the wholesale price is two-fold. First, increasing wholesale price leverages the bankruptcy threshold such that at a higher probability the retailer will file for bankruptcy. The increasing wholesale price drives down the selling quantity of the retailer and thereby increases the expected retail price. On the other hand, when the variance of demand shock is low, the supplier has no room to charge a higher wholesale price and therefore the wholesale price equals to the benchmark.

Corollary 3.4. *In the bankruptcy risk range of (N, N) equilibrium, i.e., $\bar{z} \geq \hat{\gamma} A$, the profit of the retailer is always lower than in that the no bankruptcy risk range.*

As the Stackelberg followers, retailers earn less profit than in the no bankruptcy risk range. When retailers are symmetric with respect to their financial status, they have no incentive to strategically leverage the trade credit offered by the supplier. In other words, they would prefer to use their initial capital.

3.4.3 Equilibrium of (N, Y)

In this subsection, we investigate the equilibrium when the financial statuses of the retailers are different. Without lose of generality, we assume that retailer 1 has no initial capital while retailer 2 has sufficient initial capital. The problem is also analyzed by backward induction. At the second stage, given the wholesale price and trade credit interest rate, retailers determine the selling quantities without knowing the market condition. Based on the former analysis, we can readily write down the decision problems of the retailers. Retailer 1's expected profit is given by the following piece-wise function.

$$\pi_1^{NY}(q_1) = \begin{cases} \int_{\hat{z}_1}^{\bar{z}} ((A - q_1 - \gamma q_2 + z_1)q_1 - w_1 q_1(1 + r_1)) f(z_1) dz_1 \\ \qquad\qquad\qquad\qquad \text{if } q_1 \geq A - \bar{z} - \gamma q_2 - w_1(1 + r_1) \\ (A - q_1 - \gamma q_2)q_1 - w_1 q_1(1 + r_1) \\ \qquad\qquad\qquad\qquad \text{if } q_1 < A - \bar{z} - \gamma q_2 - w_1(1 + r_1) \end{cases}$$

The critical threshold for retailer 1, $\hat{z}_1$, is defined by the break-even condition $(A - q_1 - \gamma q_2 + \hat{z}_1)q_1 = w_1 q_1(1 + r_1)$. On the other hand, retailer 2, with sufficient initial capital, determines the selling quantity to maximize the expected profit as follows:

$$\pi_2^{NY} = (A - q_2 - \gamma q_1)q_2 - w_2 q_2$$

Therefore, the best response function of retailer 1 is derived from the first-order condition:

$$q_1^{NY}(q_2) = \begin{cases} \frac{1}{3}(A - w_1(1+r) - \gamma q_2 + \bar{z}) & \text{if } q_2 \geq \frac{A - w_1(1+r) - 2\bar{z}}{\gamma} \\ \frac{1}{2}(A - w_1(1+r) - \gamma q_2) & \text{if } q_2 < \frac{A - w_1(1+r) - 2\bar{z}}{\gamma} \end{cases}$$

Retailer 2's response function is also given as

$$q_2^{NY}(q_1) = \frac{1}{2}(A - w_2 - \gamma q_1)$$

The response function of the retailer with no initial capital is a piece-wise function depending on the selling quantity of the competitor. However, the response function of the retailer with sufficient initial capital is a simple function of competitor's selling quantity. When retailer 2's selling quantity is larger than the threshold $\frac{A - w_1(1+r) - 2\bar{z}}{\gamma}$, the selling quantity of retailer 1 is biased by the demand uncertainty. Otherwise, the response functions of the two retailers are the same. The intersection of the best response functions gives the second-stage equilibrium. The selling quantity of the retailer with no initial capital is given by

$$q_1^{NY}(w_1, w_2, r) = \begin{cases} \frac{(2-\gamma)A - 2w_1(1+r) + \gamma w_2 + 2\bar{z}}{6-\gamma^2} & \text{if } \bar{z} \geq \frac{(2-\gamma)A - 2w_1(1+r) + \gamma w_2}{4-\gamma^2} \\ \frac{(2-\gamma)A - 2w_1(1+r) + \gamma w_2}{4-\gamma^2} & \text{if } \bar{z} < \frac{(2-\gamma)A - 2w_1(1+r) + \gamma w_2}{4-\gamma^2} \end{cases}$$

while the selling quantity of retailer with sufficient capital is given by

$$q_2^{NY}(w_1,w_2,r)=\begin{cases}\frac{(3-\gamma)A+\gamma w_1(1+r)-3w_2-\gamma\bar{z}}{6-\gamma^2} & \text{if } \bar{z}\geq\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}\\ \frac{(2-\gamma)A+\gamma w_1(1+r)-2w_2}{4-\gamma^2} & \text{if } \bar{z}<\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}\end{cases}$$

Depending on the variance of demand uncertainty, the selling quantities of the retailers are all piecewise functions. When the variance of demand shock is smaller than the threshold $\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}$, both retailers sell the same quantities. When the variance of demand shock is larger than the threshold, the retailer with no initial capital falls into the bankruptcy risk rang and the selling quantities of both retailers are biased by the variance of demand shock. For the retailer with no initial capital, the effect of demand shock directly stems from the bankruptcy threshold. While for the retailer with sufficient capital, the influence of the demand shock is secondary effect from the competitor's selling quantity. Moreover, the influence direction of demand uncertainty is reverse for two retailers. With a larger variance of demand shock, the retailer with no initial capital sells more while the retailer with sufficient capital sells less. Substituting the retailer's profit function, the expected profit of retailer with no initial capital is given by,

$$\pi_1^{NY}(w_1,w_2,r)=\begin{cases}\frac{((2-\gamma)A-2w_1(1+r)+\gamma w_2+2\bar{z})^3}{(6-\gamma^2)^3\bar{z}} & \text{if } \bar{z}\geq\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}\\ \frac{((2-\gamma)A-2w_1(1+r)+\gamma w_2)^2}{(4-\gamma^2)^2} & \text{if } \bar{z}<\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}\end{cases}$$

The expected profit of retailer with sufficient initial capital is given by,

$$\pi_2^{NY}(w_1,w_2,r)=\begin{cases}\frac{((3-\gamma)A+\gamma w_1(1+r)-3w_2-\gamma\bar{z})^2}{(6-\gamma^2)^2} & \text{if } \bar{z}\geq\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}\\ \frac{((2-\gamma)A+\gamma w_1(1+r)-2w_2)^2}{(4-\gamma^2)^2} & \text{if } \bar{z}<\frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2}\end{cases}$$

Anticipating the second-stage equilibrium, supplier determines the wholesale prices and trade credit interest rate in the first stage to maximize the expected profit.

$$\pi_s^{NY}(w_1,r,w_2)=\begin{cases}\int_{\underline{z}}^{\hat{z}_1}((A-q_1-\gamma q_2+z)q_1)f(z_1)dz_1+\int_{\hat{z}_1}^{\bar{z}}w_1(1+r)q_1f(z_1)dz_1+w_2q_2 \\ \qquad \text{if } 2w_1(1+r)-\gamma w_2\geq(2-\gamma)A-(4-\gamma^2)\bar{z} \\ w_1(1+r)q_1+w_2q_2 \\ \qquad \text{if } 2w_1(1+r)-\gamma w_2<(2-\gamma)A-(4-\gamma^2)\bar{z}\end{cases}$$

Solving the problem, we can derive the wholesale prices and trade credit interest rate in equilibrium. The following proposition summarizes the equilibrium results.

Proposition 3.5. *When retailer 1 has no initial capital and retailer 2 has sufficient initial capital, the equilibrium wholesale prices and the trade credit interest rate are given by*

$$w_1^{NY}(1+r^{NY})=\begin{cases}\frac{1}{12}\left(3(4-\gamma)A+(24-8\gamma^2+\gamma^4)\bar{z}-(6-\gamma^2)\Delta_2\right) & \textit{if } \bar{z}\geq\tilde{\gamma}A \\ \frac{A}{2} & \textit{if } \bar{z}<\tilde{\gamma}A\end{cases}$$

$$w_2^{NY}=\frac{A}{2}$$

the selling quantity of retailer with no initial capital is given by

$$q_1^{NY}=\begin{cases}\frac{1}{6}\left(\Delta_2-(2-\gamma^2)\bar{z}\right) & \textit{if } \bar{z}\geq\tilde{\gamma}A \\ \frac{A}{4+2\gamma} & \textit{if } \bar{z}<\tilde{\gamma}A\end{cases}$$

the selling quantity of retailer with sufficient initial capital is given by

$$q_2^{NY} = \begin{cases} \frac{1}{12}\left(3A + (2\gamma - \gamma^3)\bar{z} - \gamma\Delta_2\right) & \text{if } \bar{z} \geq \tilde{\gamma}A \\ \frac{A}{4+2\gamma} & \text{if } \bar{z} < \tilde{\gamma}A \end{cases}$$

Supplier's expected profit at equilibrium is given by

$$\pi_s^{NY} =$$

$$\begin{cases} \frac{1}{216}\left(27A^2 + 6A(2-\gamma)(2\Delta_2 - 3(2-\gamma^2)\bar{z}) + 2(2-\gamma^2)^2(\Delta_2 - (2-\gamma^2)\bar{z})\bar{z}\right) & \text{if } \bar{z} \geq \tilde{\gamma}A \\ \frac{A^2}{2(2+\gamma)} & \text{if } \bar{z} < \tilde{\gamma}A \end{cases}$$

Retailer with no initial capital has equilibrium profit

$$\pi_1^{NY} = \begin{cases} \frac{1}{216\bar{z}}(\Delta_2 - (2-\gamma^2)\bar{z})^3 & \text{if } \bar{z} \geq \tilde{\gamma}A \\ \frac{A^2}{4(2+\gamma)^2} & \text{if } \bar{z} < \tilde{\gamma}A \end{cases}$$

Retailer with sufficient capital has equilibrium profit

$$\pi_2^{NY} = \begin{cases} \frac{1}{144}(3A + \gamma(2-\gamma^2)\bar{z} - \gamma\Delta_2)^2 & \text{if } \bar{z} \geq \tilde{\gamma}A \\ \frac{A^2}{4(2+\gamma)^2} & \text{if } \bar{z} < \tilde{\gamma}A \end{cases}$$

where $\Delta_2 = \sqrt{\bar{z}(6A(2-\gamma) + \bar{z}(2-\gamma^2)^2)}$
and $\tilde{\gamma} = \frac{1}{4(2-\gamma^2)^2}((10-\gamma^2)\sqrt{\frac{(2-\gamma)(10-\gamma^2)}{2+\gamma}} - (2-\gamma)(14+\gamma^2))$.

The equilibrium results for the (N, Y) is in a similar to (N, N). There exists a threshold, $\tilde{\gamma}A$, for the variance of demand shock by which the results are divided into two segments. The supplier's profit is continuous with $\bar{z}$. We can further check that the critical ratio $\tilde{\gamma}$ is also decreasing with competition intensity. Hence, when the retailers have different financial status, increasing either the variance of demand shock or the intensity of the downstream competition could drive the retailer with financial shortage into bankruptcy risk range.

When the variance of demand shock is low, the situation is reduced to the benchmark. The retailer with no initial capital also has no bankruptcy risk. The financial status of the retailers does not cause any differences. The supplier charges wholesale prices as the benchmark, i.e., $A/2$. Both retailers sell the same amount of quantities as benchmark.

When the variance of demand shock is large, the retailer falls into the bankruptcy risk range. The supplier charges adjusted wholesale price which is higher than the wholesale price in benchmark to compensate for the possible bankruptcy. For the retailer with no financial status, the wholesale price is always $A/2$. Therefore, we don't observe the abetment effect which describes the phenomenon that the supplier may deliberately abet the competitor's predatory behavior towards the financially distressed retailer (Yang et al. 2015). This is because that we do not consider the intertemporal effect of bankruptcy. Bankruptcy would make the surviving retailer as a monopoly and hurt the supplier's profit in the two-period time window as Yang et al. (2015). However, we do observe the supplier coordinating two channels when the retailers have unbalanced financial status as shown later. Moreover, the competition between the retailers is also changed. The following corollary presents the comparison of the selling quantities between the retailers.

Corollary 3.6. *In the bankruptcy risk range of (N, Y) equilibrium, i.e., $\bar{z} \geq \tilde{\gamma} A$, the selling quantities of the retailers have the following relationship: when $\tilde{\gamma} A \leq \bar{z} < \frac{3A}{4(\gamma+2)}$, $q_1^{NY} < q_i^{YY} < q_2^{NY}$; when $\bar{z} \geq \frac{3A}{4(\gamma+2)}$, $q_1^{NY} \geq q_i^{YY} \geq q_2^{NY}$.*

In the bankruptcy risk range, when the variance of demand shock is less than $\frac{3A}{4(\gamma+2)}$, the retailer with sufficient capital orders more than the retailer with no initial capital, who sells less than the benchmark under this situation. This is similar to the predation effect in Yang et al. (2015). The influence of the variance of demand shock to the selling quantity of retailer 2 is the second-order effect resulting from retailer 1's selling quantity as we can see from the response function $q_2^{NY} = \frac{1}{2}(A - w_2 - \gamma q_1)$. However, when the variance of demand shock is greater than $\frac{3A}{4(\gamma+2)}$, the relationship is reversed. The retailer with no initial capital competes more aggressively and sells more than the retailer with sufficient capital under this condition.

Corollary 3.7. *In the bankruptcy risk range of (N, Y) equilibrium, i.e., $\bar{z} \geq \tilde{\gamma}A$, the profit of retailer with no initial capital is always lower than in the no bankruptcy risk range. The profit of the retailer with sufficient capital is higher than in the no bankruptcy risk range when $\tilde{\gamma}A \leq \bar{z} < \frac{3A}{4(\gamma+2)}$ and is lower otherwise.*

As we can see, the bankruptcy risk hurts the profit of the retailer with no initial capital under the trade credit scheme. On the contrary, the retailer with sufficient initial capital can obtain higher profit when $\bar{z} < \frac{3A}{4(\gamma+2)}$. However, when $\bar{z} > \frac{3A}{4(\gamma+2)}$, the profit of the retailer with sufficient capital is also hurt.

3.4.4 Comparison

First of all, we look into the problem from the dimension of competition. The results in bilateral monopoly work as a good benchmark for us to understand the effect of downstream competition. When the supplier sells through a monopolistic retailer who has financial distress, the retailer is less likely to bankrupt. The downstream competition increases the financially retailer's bankruptcy risk no matter what the competitor's financial status is. With competition, the retailers would like to sell more thereby increasing the bankruptcy lost. We also find that with downstream competition, the adjusted wholesale price is lower when the variance of demand shock is low and is higher otherwise. Therefore, the issue of double marginalization is more severe when the variance of demand shock is high.

Next, we'll look at the integrated impact of the competition and retailers' financial statuses. The equilibrium results of (N, N) and (N, Y) enable us to investigate the effect of trade credit on the combination of downstream financial statuses, demand characteristics and competition intensity. Since the equilibrium results in the no bankruptcy risk range of both (N, N) and (N, Y) are the same as the benchmark, we focus on the results in the bankruptcy risk range of (N, N) and (N, Y).

Corollary 3.8. *For any γ, the critical ratios have the following relationship: $\hat{\gamma} > \tilde{\gamma}$.*

From the earlier analysis, we know that increasing the competition intensity and the variance of demand can drive the firm into the bankruptcy risk range. The implication of the above corollary is that a comparatively lower competition intensity could drive the firms into the bankruptcy risk range in (N, Y) for a given variance of demand shock. From another perspective, for a given competition intensity, a less uncertain market could drive the firm into the bankruptcy risk range in (N, Y). The financially distressed retailer is more likely to be bankrupt when he is competing with a retailer who has sufficient capital. Moreover, the joint bankruptcy risk range of (N, N) and (N, Y) is given by $\hat{\gamma}A \leq \bar{z} < A$.

Offering trade credit, the supplier could acquire the retailer's revenue for compensation if the liquidation of bankruptcy is executed. Assuming the uncertainty risk, the supplier would charge a higher wholesale price as premium. A larger variance of demand shock gives the space for supplier to increase the adjusted wholesale price. The corollary below summarizes the comparison of adjusted wholesale prices.

Corollary 3.9. *The adjusted wholesale price of trade credit contract has the following relationship: when $\hat{\gamma}A \leq \bar{z} < A$, $w_1^{NY}(1 + r_1^{NY}) < w_i^{NN}(1 + r_i^{NN})$.*

We find that supplier charges a lower adjusted wholesale price in the (N, Y) scenario, that is the retailers have different financial status. To understand the result, we can proceed from the wholesale price of the retailer with sufficient capital, $A/2$, which is same as benchmark. Also the wholesale price $A/2$ is lower than $w_i^{NN}(1+r_i^{NN})$, as shown from Propositions 3.2 and 3.5. Therefore, compared to the scenario of (N, N), the supplier in the scenario of (N, Y) would like to offer a lower adjusted wholesale price to balance the two selling channels to obtain a higher profit. This is parallel to the bail-out effect in Yang et al. (2015). Compared to the case where both retailers are financially distressed, the supplier is likely to help the retailer with financial distress who is competing with a retailer with sufficient capital. That is, when the retailers have unbalanced financial statuses, the supplier would coordinate the two channels by lowing the price difference.

Given the wholesale price and the trade credit interest rate, retailers adjust the selling quantities accordingly. The relationship between the selling quantities also reflects the change of the retailers' competing behavior.

Corollary 3.10. *The selling quantities of the retailer have the following relationship: when* $\hat{\gamma}A \leq \bar{z} < \frac{3A}{4(2+\gamma)}$, $q_1^{NY} < q_i^{NN} < q_2^{NY}$; *when* $\frac{3A}{4(2+\gamma)} \leq \bar{z} < A$, $q_1^{NY} \geq q_i^{NN} \geq q_2^{NY}$.

The selling quantity of the retailer with no initial capital increases with the variance of demand shock. When the retailer with no initial capital competes with a retailer with a similar financial status, the selling quantity increases with the variance of demand shock in a more gentle pace. When $\bar{z}$ is less (larger) than the threshold, the production quantity of the retailer with no initial capital in (N, Y) equilibrium is smaller (larger) than in (N, N) equilibrium. When the retailers have different financial statuses, the effects of the variance of demand shock on the selling quantities are asymmetric.

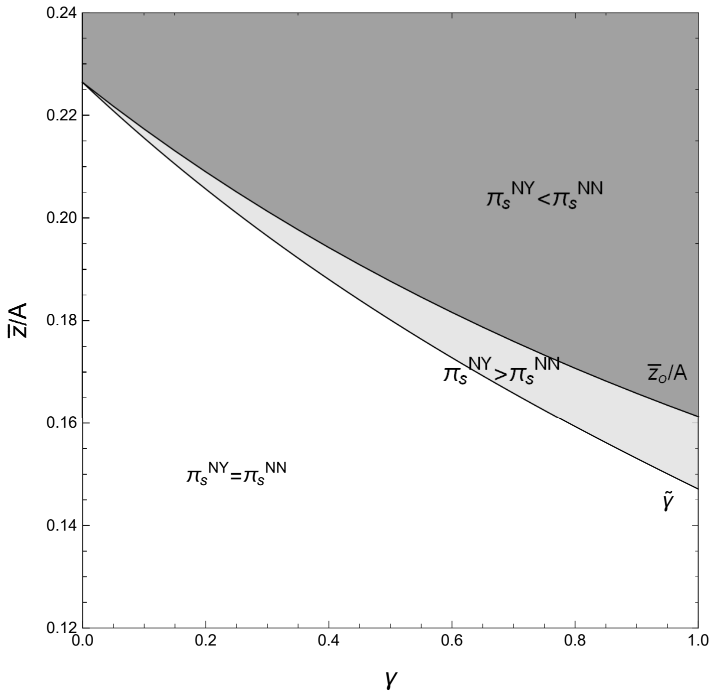


Figure 3.2: Supplier's profit

The demand uncertainty influences the supplier who offers trade credit. From Propositions 3.2 and 3.5, we know that the supplier is able to make use of the risk by the leadership power in the supply chain. By increasing the wholesale prices, the supplier could acquire more profit by assuming the demand risk. This is also a supportive evidence for the supplier's providing trade credit. The following corollary compares the supplier's profit in (N, N) and (N, Y) equilibria.

Corollary 3.11. *The supplier's profits have the following relationship:* $\exists \bar{z}_o \in (\tilde{\gamma}\bar{z}, A)$ *such that when* $\tilde{\gamma}A \leq \bar{z} < \bar{z}_o$, $\pi_s^{NY} > \pi_s^{NN}$ *and when* $\bar{z}_o < \bar{z} < A$, $\pi_s^{NY} < \pi_s^{NN}$.

Figure 3.2 depicts the comparison of the supplier's profits. The horizontal axis is the intensity of downstream competition. The vertical axis is $\bar{z}/A$, which could be understood as the standardized variance of demand shock. When the retailers in both (N, N) and (N, Y) equilibria are in the no bankruptcy risk range, the supplier gets equal profit for the different combinations of the downstream retailers' financial statuses. When the demand uncertainty is moderate, the supplier's profit from unbalanced retailer financial status is larger. When the demand uncertainty is even larger, the supplier's profit is higher when both retailers are financially distressed. This is because that the supplier always try to balance the two channels. When the retailers have unbalanced financial status, the supplier has to lower the adjusted wholesale price to decrease the difference from the retailer with sufficient capital. This channel coordination is costly for the supplier.

Looking from another perspective, for a given variance of demand shock, the supplier may firstly prefer the unbalanced retailers' financial status and then prefer the balanced retailers' financial status with the increasing of downstream competition intensity. The implication from the above result is that when the supplier is choosing downstream partners, she should consider the competition intensity, demand uncertainty and the financial status of the retailers.

The equilibria also enable us to investigate the influence of financial status on the retailers' profitability, as presented in the following two corollaries.

Corollary 3.12. *When* $0 < \bar{z} < \tilde{\gamma}A$, $\pi_1^{Nf} = \pi_1^{Yf}$, $\forall f \in \{Y, N\}$; *when* $\tilde{\gamma}A < \bar{z} < A$, $\pi_1^{Nf} < \pi_1^{Yf}$, $\forall f \in \{Y, N\}$.

Figure 3.3 depicts the result of Corollary 3.12. The horizontal and vertical axises have the same meaning as the Figure 3.2. The above result shows that the retailer obtains a higher profit with a better financial status no matter what the financial status of its competitor is, considering the bankruptcy risk. Although there's bail-out effect from the supplier when the retailers have unbalanced financial statuses, the cost of bankruptcy liquidation is harmful for the retailers. The bankruptcy effect could be decomposed to the variance of demand shock or the competition intensity. With the increasing of either the variance of demand shock and the competition intensity, the

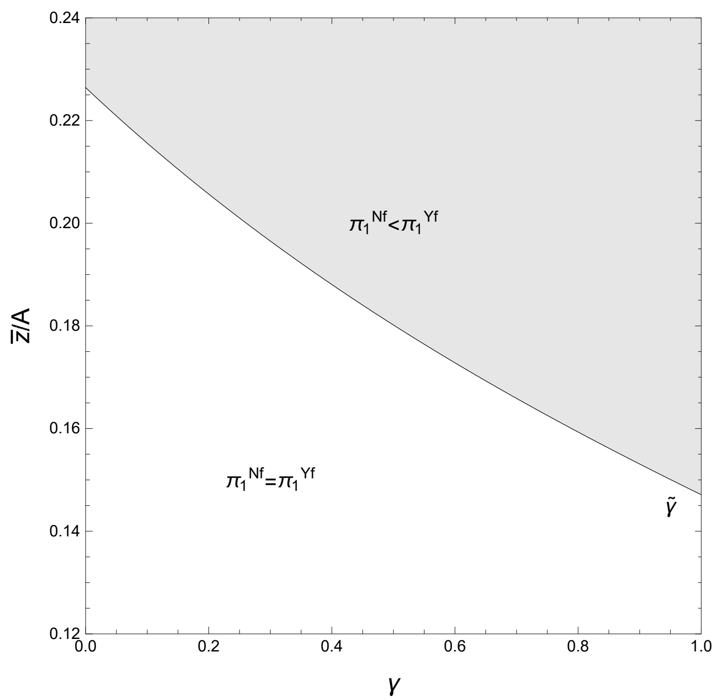


Figure 3.3: The impact of retailer's financial status

firms may be forced into the bankruptcy risk range. The following corollary summarizes the effect of the competitor's financial status.

Corollary 3.13. *When* $0 < \bar{z} < \tilde{\gamma}A$, $\pi_1^{fN} = \pi_1^{fY}$, $\forall f \in \{Y, N\}$; *when* $\tilde{\gamma}A < \bar{z} < \frac{3A}{4(2+\gamma)}$, $\pi_1^{fN} > \pi_1^{fY}$, $\forall f \in \{Y, N\}$; *when* $\frac{3A}{4(2+\gamma)} \leq \bar{z} < A$, $\pi_1^{fN} \leq \pi_1^{fY}$, $\forall f \in \{Y, N\}$.

Figure 3.4 shows the result of Corollary 3.13. The change of competitor's financial status has more subtle effect. In the bankruptcy risk range, when the variance of demand shock is moderate, the improvement of competitor's financial status is harmful for the retailer, no matter what financial status the retailer is. However, when the variance of demand shock is relatively large, the improvement of competitor's financial status is beneficial.

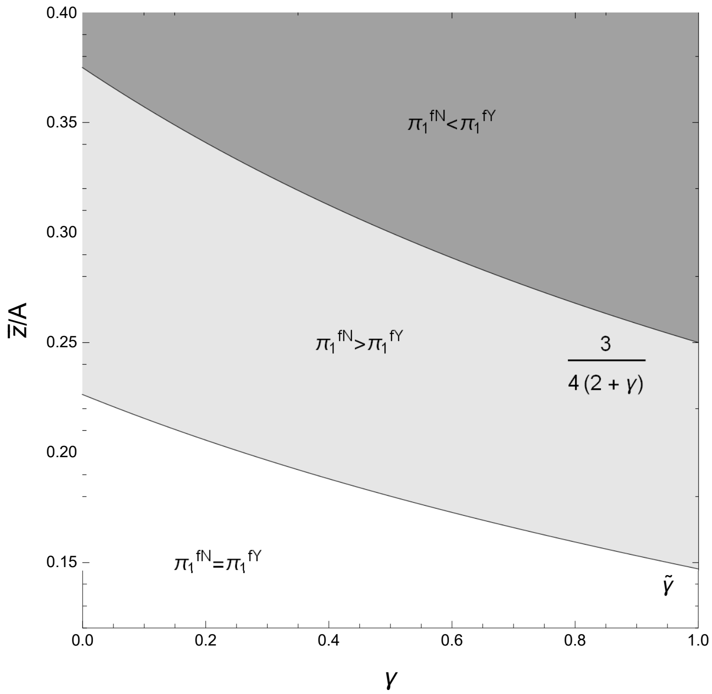


Figure 3.4: The impact of competitor's financial status

3.5 Conclusion

Trade credit is a commonly used contract in supply chain finance. It is an important resource of firms' short-term financing. In this paper, we investigate the usage and effect of trade credit considering the joint effect on the vertical and horizontal supply chain relationship. Offering trade credit to the financially distressed retailers, the supplier is influenced by the market uncertainty. We identified the change of competition behavior of the supply chain parties and the implications for their profits.

First of all, we identify the characteristics which influence the bankruptcy. Increasing the variance of demand shock and downstream competition intensity can drive the financially distressed retailer into the bankruptcy risk range. In the bankruptcy risk range, the competing behavior and profits are changed.

If the retailer is bankrupt, the supplier is compensated with the retailer's revenue. The supplier charges a higher wholesale price as premium for the market risk. However, when the retailers have unbalanced financial statuses, the supplier offers a lower adjusted wholesale price compared to the case where both retailer are financially distressed. In this way, the supplier bails out the financially distressed retailer when he is competing with the retailer who has sufficient capital. The supplier has the incentive to balance the two channels by decreasing the differences between the retailers.

From the perspective of profitability, the supplier may prefer the unbalanced retailers' financial status when the variance of demand shock is in an intermediate range and prefer the case where both retailers are financially distressed when the variance of demand shock is relatively large. This corresponds to the observation that Ocean Trawler expands sales into Togo and provides trade credit for the retailers. That is, suppliers would like extend business into markets where the retailers have poor financial strength besides the customers in traditional market.

The competition between the retailers is also changed. When both retailers are financially distressed, the selling quantities are larger than the benchmark when the variance of demand shock is relatively large. Under this condition, the competition between the retailers are intensified. We also observe predatory effect between the retailers when they have unbalanced financial status. When the variance of demand shock is moderate, the retailer with sufficient capital predates the financially distressed retailer by enlarging the selling quantity. On the other hand, the financially distressed retailer also overly competes with the retailer with sufficient capital when the variance of demand is relatively large.

Better financial status would bring a higher profit for the retailer no matter what the financial status of its competitor is. However, the change of competitor's financial status could bring either positive or negative effect on the retailer's profits. Given the intensity of competition, when the variance of demand shock is moderate, competitor's enhancement in financial status cuts the retailer's profit; when the variance of demand shock is high, the retailer's profit will increase with the improvement of its competitor's financial status.

Extending the problem into multi-period framework is an meaningful direction for future research. Future research could also incorporate the default cost or a more general distribution function.

Chapter 4

Competitive Product Recovery Strategy with Remanufacturing

4.1 Introduction

Technology advancements and public environmental awareness incubate market for the remanufactured products. Many original equipment manufacturers (OEM) take actions to integrate remanufacturing to exploit the increasingly lucrative market. The value of U.S. remanufactured production grew by 15 percent to at least $43.0 billion from 2009 to 2011, supporting 180,000 full-time U.S. jobs (USITC 2012). A pilot program of auto remanufacturing in China is estimated to generate output value worth 2 billion yuan (US$300 million). In addition to the great economic potential, remanufacturing is proved to be effective tactics to leverage the secondary market, prevent industry entrant, and segment market for price discrimination, etc. However, conducting such major operations as remanufacturing ask for the massive interaction and coordination between supply chain members. The traditional supply chain management asks for incorporating the operations of the reverse channel. Closed-loop supply chain management attracts keen attention of both academia and industry.

Remanufacturing refers to a series of activities to rebuild, repair, and restore the product to as-new condition. The very fundamental step of remanufacturing process is product recovery through which lots of interactions between forward and reverse channel are intensified since the used products are reversely circulated back to the manufacturer as production material. (Throughout this paper, we use product collection and product recovery interchangeably). Product recovery calls for many interactions between parties in the channel.

In reality, there're two main product recovery modes. The manufacturer may directly collect used products from consumers or appoint retailer to collect. For example, Xerox, a pioneer in remanufacturing, has famous manufacturer take-back program. Their green remanufacturing program produces high-quality copier by reusing the collected used products. Likewise, the Lenovo, Epson and many other manufacturers offer free mail back programs to collect the used products.

On the other hand, we also observe that manufacturer adopts the alternative product recovery mode, i.e., the retailer is assigned to collect the used products for the manufacturer. For example, Caterpillar, a famous large-scale machinery manufacturer, has proactively engaged with dealers for product take back. PC manufacturers like HP also assign retailer to collect the used products.

One implication from the product recovery is that when manufacturer utilizes retailer to collect used products, the retailer is endowed with a dual role of material supplier who could influence the cost side of the supply chain and marketing agent who could influence the price side as well. Whereas inserting an independent profit-maximizing retailer as an intermediary was initially studied in marketing literature which primarily concern about the pricing side issue. Although it is proved that retailer could help buffer manufacturer from price competition when the product substitutability is high (McGuire and Staelin 1983), whether the manufacturer would like to insert retailer in the reverse channel is uncertain.

Given that remanufacturing plays an important role in the competitive market and product recovery is a fundamental step of remanufacturing system, this paper investigate the following problems: what's the product recovery strategy in equilibrium for the competing supply chains? Is the equilibrium Pareto efficient? What are the effect of competition intensity, product collection efficiency, and power structure of the supply chain on equilibrium?

To develop an in-depth understanding of these important questions, we consider a two-stage game in an industry with two competing manufacturers who sell substitutable products through their exclusive independent retailers. The products could be produced by the raw material or remanufactured by the collected used parts. Customers are indifferent with the products and the remanufactured products. In the first stage, manufacturers simultaneously determine the product recovery strategy, i.e. collect by themselves (direct recovery) or by retailers (indirect recovery). The agent who bears the collection duty would incur a collection-rate-related investment cost. Four possible strategy combinations are formed: $(Direct, Direct)$, $(Indirect, Indirect)$, $(Direct, Indirect)$, and $(Indirect, Direct)$. The first word in each bracket represents the policy adopted by supply chain 1 and the second represents supply chain 2's policy. Based on the strategy combinations formed, manufacturer and retailer in the second stage make a decision of the pricing and collection rate following three types of game sequence: Stackelberg - manufacturer as the leader, Stackelberg - retailer as the leader and vertical Nash.

Our main results shows that both inter- and intra-channel factors are influencing the equilibrium. Power structure stands for the intra-channel factor while the competition intensity stands for the inter-channel factor. When the manufacturer and retailer are engaged in vertical Nash, the equilibrium is invariant to the channel competition. Without channel power, the manufacturer charges equal margin with the retailer and the retail prices are the same no matter what product recovery strategy is adopted. Manufacturers prefer to let retailer to collect the products so that he would not bear the related cost. Under this situation, (*Indirect, Indirect*) is the unique Nash equilibrium and is also Pareto efficient.

When there's leadership in the supply chain, multiple equilibria may occur. Both direct recovery and indirect recovery may be chosen in the equilibrium when the competition intensity and effective ratio of collection are high. Specifically, when the manufacturer is the Stackelberg-leader, the collection rate under direct recovery mode is lower, because the manufacturer is less effective to improve the collection rate. The larger cost saving from higher collection rate can only be partially reflected on the retail price due to the issue of double marginalization. Moreover, with indirect recovery strategy, the supply chain charges lower prices and the manufacturer could obtain higher profit given the competitor's price. Therefore, (*Indirect, Indirect*) is a prevailing equilibrium when manufacturer is the Stackelberg leader. However, when the competition intensity and the effective ratio of collection are high, (*Direct, Direct*) is also Nash equilibrium and is Pareto efficient. This is because when the competition intensity is high and efficiency of product collection is high, the higher retail price under direct recovery strategy can avoid over competition between the supply chains and achieve higher profits for the manufacturers.

When the retailer is the Stackelberg-leader, the manufacturer prefers to collect the used products by himself. The manufacturer is more incentivized to increase the collection rate so that the average production cost and thereby the retail price are lower. Therefore, (*Direct, Direct*) is the prevailing Nash equilibrium. However, with direct recovery strategy, the manufacturers might be trapped into prisoners' dilemma when either the competition intensity is high or the effective ratio of collection is low. (*Indirect, Indirect*) may be Nash equilibrium when the competition intensity and effective ratio of collection are high.

In sum, the firm with channel power has less incentive to enlarge the collection rate. Higher collection rate reduces the average production cost so that the supply chain is able to charge lower price in the market. At equilibrium, manufacturers would adopt the product recovery strategy which can achieve lower price. However, manufacturers may be trapped into prisoners' dilemma with the strategy. On the other hand, the product recovery strategy which leads to higher retail price can turn to be a Pareto efficient Nash equilibrium when the competition intensity and effective ratio of collection are high.

As noted, Savaskan et al. (2004) find that in bilateral monopoly channel with one manufacturer and one retailer, the retailer, being the agent closer to the customers, has dominant advantage to collect the used products. That is, when there's no competition considered, manufacturer would like to insert retailer in the reverse channel. However, the above results show that with competition both direct and indirect strategies could be the equilibrium. In another word, competition implicates the interaction between the parties within a supply chain.

The remainder of this chapter is organized as follows. In section 4.2, we review related literature on remanufacturing and closed-loop supply chain. Section 4.3 is devoted to introduce the model formulation where we describe the structure of the supply chain and the game sequence when two competing supply chain make the decisions about product recovery. In section 4.4, we analyze the equilibrium of product recovery. The conclusion and future research directions are outlined in section 4.5.

4.2 Literature

With the development of the environment awareness, the research on remanufacturing and product recovery is an emerging area and undergoing dramatic growth. Remanufacturing shows great economic potential and shapes the pervasive aspects of supply chain management, also known as green supply chain or closed-loop supply chain. We refer to Atasu et al. (2008), Guide and Van Wassenhove (2009), Tang and Zhou (2012) and Souza (2013) for comprehensive review.

In addition to the environmental importance and cost saving purpose, remanufacturing could be used as a powerful competition strategy. Ferguson and Toktay (2006) find that the remanufacturing could be used as entry-deterrent strategy. In a different setting, Atasu et al. (2008) claim that remanufacturing is more beneficial under competition than in a monopoly setting. Oraiopoulos et al. (2012) study how the strategy of secondary market shaped by competitive advantage, product characteristics, and consumer preferences. The secondary markets is where the used or remanufactured products are traded.

Product recovery, a crucial segment of remanufacturing system, also receives lots of attention. Referring to product recovery, it contains the decisions about how and how much to collect. With the perfection of legislation, some researchers study from the perspective of compliance of the legislation, such as Atasu et al. (2013) and Esenduran and Kemahlıoğlu-Ziya (2015). Within supply chain firms, the strategic influence of product recovery strategy lies in the operations decision interaction between the forward channel and reverse channel. Savaskan et al.(2004) consider the product recovery in bilateral monopoly and point out that retailers are more effective to assume the collection work. In a subsequent serial research, Savaskan and Wassenhove (2006) consider the problem in a more complicated supply chain where the manufacturer sells product through two competing retailers. The competition, however, is restricted in the retail market level. Miao et al. (2017) investigate the influence of using trade-in to on the product recovery strategy. Since product recovery is an important part in remanufacturing system and remanfuacturing has strategic importance in competition, we aim to understand how the product recovery strategy would be influenced in the competing supply chains. The competition nowadays usually involve the chain to chain competition especially on the big issues like product recovery. On the other hand, we would like to investigate how would the product recovery influence the operations decisions.

Our work is also related to the design of distribution channel. McGuire and Staelin (1983) is the seminal paper which studies the issue of integration in the distribution channel. Manufacturer could choose to sell directly to the consumers or insert an independent profit-maximizer inbetween. In the subsequent research Moorthy (1988) study the channel-structure problem with the strategic interaction with competitors. They find that the coupling of demand dependence and strategic dependence determine the equilibrium. When retailer collects the used products, it is equivalent that the manufacturer decentralizes in the reverse channel.

Our paper also relates to the literature on power structure in supply chain. The power structure could be characterized through different sequence of action made by each party within the supply chain (Fader and Moorthy 2012, Choi 1991). The first mover, anticipating the response of the second mover, is generally regarded to be the leader (Shi et al. 2013). The second mover can only take action after observing the decision of the first mover. In the common two-echelon supply chain, both manufacturer and retailer could function as the leader: Manufacturer Stackelberg-leader (McGuire and Staelin 1983, Lariviere and Porteus 2001) and Retailer Stackelberg-leader (Raju and Zhang 2005, Geylani et al. 2007). When there's no apparent power difference, two firms would move simultaneously. Then, the Nash equilibrium would be derived from the intersection of their best response function (Jeuland and Shugan 1983). Various problems are investigated under the impact of power structure, such as channel structure (Choi 1991), market price and profits (Ertek and Griffin 2002), ordering time (Ferguson 2003, Ferguson et al. 2005), supply chain performance (Majumder and Srinivasan 2006, Shi et al. 2013) and supplier alliances in a assembly system (Nagarajan and Sošić 2009). To the best of our knowledge, we're the first to address the issue of supply chain power structure in the research of product recovery.

4.3 Model Formulation

Our primary interest is to understand the joint intra- and inter-channel implication of product recovery. To this end, we model the problem in two competing supply chains (denoted by subscript i, $i \in \{1, 2\}$), each consisting of one manufacturer (M_i) and one retailer (R_i). This is the most skylized supply chain structure to study our problem.

In each supply chain, manufacturer produces at unit cost c and sells to the retailer at wholesale price w_i. Retailer then sells to customers at retail price p_i. Customers are identical and price sensitive. Market demand is linear in the product's price and the competitor's price, $D_i(p_i, p_{3-i}) = \mu - p_i + \beta p_{3-i}$, $\mu > 0$ and $0 < \beta < 1$. Hence, the market demand decreases with the product's price and increases with competitor's price. Here μ is the base market size for each supply chain. We assume that two supply chains are faced with identical base market size. β depicts the substitution level of products by two supply chains, which is the measurement of the competition intensity. Figure 4.1 illustrates the supply chain structure of our model.

Manufacturers have built in remanufacturing system so that they could produce their products with both raw materials and collected used parts. The remanufacturing technology is to such extent that the products remanufactured by used parts function

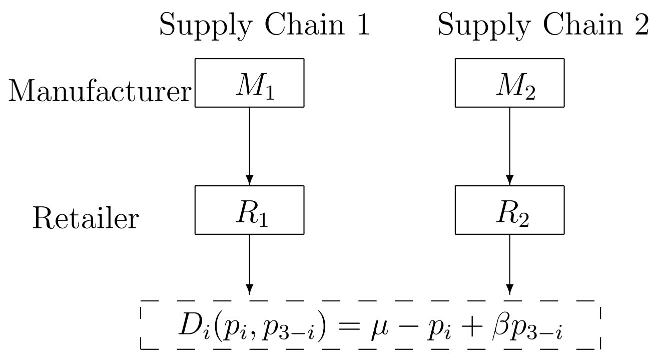


Figure 4.1: Supply chain structure

comparably as new products. Customers treat the new and remanufactured products indifferently. This assumption is commonly seen in literature (eg. Savaskan et al. 2004 and Savaskan and Wassenhove 2006). With the development of remanufacturing technology and deep-rooted environmental protection awareness, increasing amount of customers begin to show this inclination.

Remanufacturing is cost-effective and incurs unit marginal cost of c_r, $c_r < c$. The cost benefit of remanufacturing is characterized by Δ, $\Delta = c - c_r$. The cost c_r has already considered other related costs, such as buy-back payment for customers, transportation fee and inspection cost. To purify the analysis, we focus on the case of symmetric supply chains. That is, the two competing supply chains basically demonstrate comparatively similar characteristics in terms of market size, manufacturing and remanufacuring cost.

Product recovery is crucial to the remanufacturing process. We use collection rate τ_i, $0 < \tau_i < 1$, as the measurement of the production collection performance. The collection rate is defined as the proportion of current production batch which is supported by the collected used parts. The formulation is following the trend of Savaskan et al. (2004). Our research is constrained in one-period time frame and we assume that the products are in mature and steady state. The collection rate τ_i could be viewed as the response of consumers to the effort of the agent paid to the product recovery. To sustain the collection rate, a convex investment cost $B\tau_i^2$ is incurred. B is the scale parameter to characterize the efficiency of remanufacturing investment. In other words, B reflects the diminishing scale of economies of investment in product recovery. The quadratic investment cost is also seen in research on R&D, advertisement, etc. To ensure the interior solutions in all equilibriums, we impose the condition that $0 < B < 3.5\Delta^2$.

Hence, the average unit manufacturing cost could be calculated as $c - \Delta\tau_i$. We assume that each manufacturer can only re-manufacture the collected products originally produced by himself. In each supply chain, manufacturer chooses the collection mode. There're two options of collection mode. The first one is known as *direct* collection (denoted as D), where manufacturer collects used products from consumers by themselves. The other is *indirect* collection (denoted as I), where retailer is assigned to collect used products for remanufacturing. The fundamental responsibility of the agent who assumes the collection activities is to determine the collection rate τ_i and bear the collection-related investment cost $B\tau_i^2$. That is, when manufacturer adopts direct collection, he determines the collection rate and incurs the product collection investment cost. When manufacturer adopts indirect collection mode, retailer determines the collection rate and manufacturer pays unit buyback cost b_i to the retailer to acquire the used parts.

We're interested in how the supply chains' competitive behaviors would influence the equilibrium collection mode and the effect of collection mode on the firms' operations decisions. To this end, we model the problem in a two-stage game. In the first stage, manufacturers determine the product recovery strategy simultaneously. We do not consider mixed strategy equilibrium. The commitment of collection mode is reliable, because it is a mid-to-long term infrastructure investment for an organization to adjust in information systems, facility procurement and organizational structures, etc. Based on the collection mode each supply chain adopts, four possible strategy combinations as shown in Table 1 are formed: $(Direct, Direct)$, $(Indirect, Indirect)$, $(Direct, Indirect)$, and $(Indirect, Direct)$. The first word in each bracket represents the policy adopted by supply chain 1 and the second represents supply chain 2's policy. Since $(Direct, Indirect)$ and $(Indirect, Direct)$ are symmetric, we only focus on the first three scenarios, i.e. $(Direct, Direct)$, $(Indirect, Indirect)$ and $(Direct, Indirect)$. The superscript $j \in \{DD, II, DI\}$ will be used to represent the corresponding scenario in the following discussions.

	Direct	Indirect
Direct	D-D	D-I
Indirect	I-D	I-I

Table 4.1: Strategy matrix

In the second stage, each firm determines operations decisions including retail price, wholesale price and collection rate in accordance with the collection mode determined in the first stage. We consider three kinds of power structure within either supply chain: Stackelberg-manufacturer as the leader, Stackelberg-retailer as the leader and vertical Nash. Following the literature, we model the power structure through the different sequence of decision made by each party (Fader and Moorthy 2012, Choi 1991). The leader, anticipating the response of the follower, determines his decisions firstly. The follower can only take action after observing the decision of the leader. When manufacturer and retailer engage in Nash game, the equilibrium would be derived from the intersection of their best response function.

We use π_i^j to denote the profit of retailer i under scenario j and Π_i^j to denote manufacturer i's profit under scenario j. Besides, we'll use notations ‘ ¯ ’ , ‘ ˜ ’ and ‘ ˆ ’ to differentiate the equilibrium results of Stackelberg - manufacturer as the leader, Stackelberg - retailer as leader and vertical Nash respectively. We do not consider the problem of information asymmetry. Each firm has complete information about the decision. However, the information of firm's decision is confined within either supply chain. For example, when manufacturer is the Stackelberg leader and uses direct collection, retailer from the same supply chain has full information about the wholesale price and collection rate but the retailer from competing supply chain does not know.

4.4 Analysis

In this section, we'll analyze the equilibrium of product recovery under each game sequence: Stackelberg - Manufacturer as the leader, Stackelberg - Retailer as the leader and vertical Nash.

Consistent with the extensive marketing research, we substitute retail prices and wholesale prices with the retail margins and wholesale margins, representing by m_i and M_i respectively. When the supply chain uses direct collection, the wholesale price can be written as the sum of average unit production cost and wholesale margin, $w_i = c - \Delta\tau_i + M_i$. The retail price is the sum of wholesale price and retail margin, $p_i = c - \Delta\tau_i + M_i + m_i$. When the supply chain adopts indirect collection mode, retailer is responsible to collect products. Manufacturer now needs to pay a unit buyback payment to the retailer. The average unit cost is adjusted with the buyback price and the wholesale price is, $w_i = c - (\Delta - b_i)\tau_i + M_i$ and the retail price is $p_i = c - (\Delta - b_i)\tau_i + M_i + m_i$.

Specifically, when the supply chain adopts direct collection mode, retailer determines retail margin and manufacturer decides the wholesale margin and collection rate. Retailer's profit function is given by

$$\max_{m_i} \left(\left(\mu - (c - \Delta\tau_i + M_i + m_i) + \beta p_{3-i} \right) m_i \right), \tag{4.1}$$

and manufacturer's profit function is given by

$$\max_{M_i,\tau_i} \left(\left(\mu - (c - \Delta\tau_i + M_i + m_i) + \beta p_{3-i} \right) M_i - B\tau_i^2 \right). \tag{4.2}$$

When the supply chain adopts indirect collection mode, retailer determines both retail price margin and collection rate. Manufacturer determines wholesale price margin. Retailer's profit function can be expressed as

$$\max_{m_i,\tau_i} \left((\mu - (c + M_i + m_i - (\Delta - b_i)\tau_i) + \beta p_{3-i})(m_i + b_i\tau_i) - B\tau_i^2 \right), \tag{4.3}$$

and manufacturer's profit function is given by

$$\max_{M_i,b_i} \left((\mu - (c + M_i + m_i - (\Delta - b_i)\tau_i) + \beta p_{3-i}) M_i \right). \tag{4.4}$$

The problem could be solved by backward induction. In the second stage, the pricing power determines the decision sequence within each supply chain. Given the strategy combination, the operations decision of each firm is represented as a function of the competitors' decisions. The second-stage equilibrium is then derived by the intersection of the response functions of two supply chains.

In the first stage, manufacturers determine the collection mode simultaneously. The expected profit of manufacturer in the second stage gives the payoff of the strategy matrix in the first stage. We can identify the first-stage Nash equilibrium following standard procedure.

4.4.1 Stackelberg - Manufacturer as the leader

When manufacturers and retailers are engaged in Stackelberg game at the second stage and manufacturers function as the leader, manufactures make decisions anticipating retailers' responses.

Second-stage Analysis

Firms in each supply chain would firstly determine the operations decision in accordance with the product recovery strategy, taking the retail price of competing supply chain as given. When the supply chain adopts direct collection, retailer determines the retail margin given the wholesale margin and collection rate by maximizing (4.1). The best response is derived from the first-order condition, $\bar{m}_i^D(\bar{M}_i^D, \bar{\tau}_i^D, p_{3-i}) = \frac{1}{2}(\mu - (c - \Delta\bar{\tau}_i^D + \bar{M}_i^D) + \beta p_{3-i})$. Manufacturer, anticipating the response of retailer, determines wholesale margin and collection rate by maximizing (4.2). The response functions are represented as the functions of competitor's price:

$$\bar{M}_i^D(p_{3-i}) = \frac{4B(\mu - c + \beta p_{3-i})}{8B - \Delta^2} \text{ and } \bar{\tau}_i^D(p_{3-i}) = \frac{\Delta(\mu - c + \beta p_{3-i})}{8B - \Delta^2}. \tag{4.5}$$

Substituting into the retail margin we can obtain:

$$\bar{m}_i^D(p_{3-i}) = \frac{2B(\mu - c + \beta p_{3-i})}{8B - \Delta^2}. \tag{4.6}$$

The resulting retail price is then $\bar{p}_i^D(p_{3-i}) = \frac{2Bc+(6B-\Delta^2)(\mu+\beta p_{3-i})}{8B-\Delta^2}$. We can also calculate the expected profit of manufacturer as $\bar{\Pi}_i^D(p_{3-i}) = \frac{B(\mu-c+\beta p_{3-i})^2}{8B-\Delta^2}$.

When the supply chain adopts indirect collection, retailer, given the wholesale margin and buyback price, determines the retail margin and collection rate by maximizing (4.3). From first-order condition, we can get the response of retail margin and collection rate as: $\bar{m}_i^I(\bar{M}_i^I, \bar{b}_i^I, \bar{\tau}_i^I, p_{3-i}) = \frac{(2B-\bar{b}_i\Delta)(\mu-c-\bar{M}_i^I+\beta p_{3-i})}{4B-\Delta^2}$ and $\bar{\tau}_i^I(\bar{M}_i^I, \bar{\tau}_i^I, p_{3-i}) = \frac{\Delta(\mu-c-\bar{M}_i^I+\beta p_{3-i})}{4B-\Delta^2}$. The collection rate is independent with unit buyback price b_i. Therefore, the buyback payment has no incentive effect on the product recovery behavior.

Substituting $\bar{m}_i^I(\bar{M}_i^I, \bar{b}_i^I, \bar{\tau}_i^I, p_{3-i})$ and $\bar{\tau}_i^I(\bar{M}_i^I, \bar{\tau}_i^I, p_{3-i})$ into (4.4), we can obtain the objective function of manufacturer as follows:

$$\max_{\bar{M}_i^I} \frac{2B\bar{M}_i^I(\mu - c - \bar{M}_i^I + \beta p_{3-i})}{4B - \Delta^2}.$$

An observation from the manufacturer's profit function is that the buyback payment for retailer $\bar{b}_i$ is canceled out. That is to say manufacturer could achieve the optimal profit by simply adjust wholesale margin. The best response of wholesale margin is derived by the first-order condition as follows:

$$\bar{M}_i^I(p_{3-i}) = \frac{1}{2}(\mu - c + \beta p_{3-i}). \tag{4.7}$$

Substituting $\bar{M}_i^I(p_{3-i})$ into the retail margin and collection rate, we can obtain the retail margin and collection rate,

$$\bar{m}_i^I(p_{3-i}) = \frac{(2B - b_i\Delta)(\mu - c + \beta p_{3-i})}{2(4B - \Delta^2)} \text{ and } \bar{\tau}_i{}^I(p_{3-i}) = \frac{\Delta(\mu - c + \beta p_{3-i})}{2(4B - \Delta^2)}. \tag{4.8}$$

Although the retail margin depends on the buyback payment $\bar{b}_i$, the resulting retail price is given by $\bar{p}_i^I(p_{3-i}) = \frac{Bc+(3B-\Delta^2)(\mu+\beta p_{3-i})}{4B-\Delta^2}$, which is independent with the buyback payment. Manufacturer provides unit payment as seemingly compensation for product recovery but retailer has to exclude it when determining the retail margin. The buyback payment $\bar{b}_i$ functions as dummy variable. Neither the profit of manufacturer or the profit of retailer is influenced by the buyback payment. Therefore, we assume that $\bar{b}_i = 0$. The expected profit of manufacturer is calculated as $\bar{\Pi}_i^I(p_{3-i}) = \frac{B(\mu - c + \beta p_{3-i})^2}{2(4B - \Delta^2)}$.

The intersection of response functions of supply chains with direct collection mode ((4.5) and (4.6)) gives the second-stage equilibrium results of $(Direct, Direct)$ scenario. The intersection of response functions of one supply chain with direct collection mode and one supply chain with indirect collection mode ((4.5), (4.6), (4.7) and (4.8)) gives the equilibrium results of $(Direct, Indirect)$ scenario. Likewise, the equilibrium results of $(Indirect, Indirect)$ scenario can be derived by the intersection of response functions of two supply chain with indirect collection mode. All the results of operations decisions and equilibrium profits are tabulated in the appendix.

Before we proceed to analyze the first-stage equilibrium, we could get a primitive image from the comparison of responses of direct collection and indirect collection supply chains. The following lemma gives the order of collection rates and retail prices.

Lemma 4.1. *When manufacturer is the Stackelberg leader, given the retail price of competing supply chain, supply chain adopting direct collection mode collects less,* $\bar{\tau}_i^D(p_{3-i}) < \bar{\tau}_i^I(p_{3-i})$*, but charges a higher retail price,* $\bar{p}_i^D(p_{3-i}) > \bar{p}_i^I(p_{3-i})$.

These results mirror the findings from Savaskan et al. (2004). Due to double marginalization, the unit saving from remanufacturing can only partially reflect on the retail price when manufacturer collects the used products. Therefore, when the retail price of competing supply chain is exogenously fixed, the supply chain with indirect collection mode executes at a higher collection rate. Also supply chain with direct collection mode charges a higher retail price than the one with indirect collection mode. A higher price implies a lower demand. Combing the two facets, it is obvious that cost saving from remanufacturing from direct collection supply chain is less than that from indirect collection supply chain. The following lemma gives the order of manufacturers' margins and profits given the competitor's price.

Lemma 4.2. *When manufacturer is the Stackelberg leader, given the retail price of competing supply chain, manufacturer under direct collection mode has a higher margin,* $\bar{M}_i^D(p_{3-i}) > \bar{M}_i^I(p_{3-i})$*, but obtains a lower profit,* $\bar{\Pi}_i^I(p_{3-i}) > \bar{\Pi}_i^D(p_{3-i})$*.*

Given the retail price of competing supply chain, manufacturer charges higher margin when he collects used products directly. However, with a higher demand and free duty of product recovery, manufacturer could secure a higher profit with indirect collection mode. As Savaskan et al. (2004) shown, when manufacturer and retailer are bilateral monopoly, indirect collection mode is preferable for manufacturer. Considering the strategic response of competing supply chain, whether indirect collection could maintain advantage is not immediately clear.

First-stage Analysis

To investigate the first-stage Nash equilibrium, we define the concept of *effective ratio of collection* denoted by $n = \Delta^2/B$ to facilitate our analysis. Recall that Δ is the unit cost advantage from remanufacturing and B reflects the scale of diminishing returns of product collection investment. The higher level of n, the more efficient the investment to product collection. The available range for n is $0 < n < 3.5$. This condition is derived to guarantee the non-negativity of decision variable.

Adopting indirect collection mode is equivalent to that manufacturer decentralizes in the reverse channel. McGuire and Staelin (1983) study the decentralization incentive in the forward distribution channel and find that when the intensity of competition between supply chains is high, manufacturers prefer to decentralize by inserting an independent profit maximizing retailer as "buffer" to relieve competition. In the closed-loop supply chain with remanufacturing, decentralizing in the reverse channel has opposite meaning. As Lemma 4.1 shows that when retailer assumes the duty of product recovery, he collects with a higher collection rate and the retail price is lower at any given rice of competitor. Hence, the competition is exacerbated by decentralizing in the reverse channel. The following proposition characterizes the first-stage equilibrium for product recovery of competing supply chains.

Proposition 4.3. *When manufacturer is the second-stage Stackelberg-leader,*
(1) (Indirect, Indirect) is always Nash Equilibrium;
(2) (Direct, Direct) is Nash Equilibrium when $6-2\sqrt{2}<n<3.5$ *and* $\beta_1(n)<\beta<1$*.*

Both $(Direct, Direct)$ and $(Indirect, Indirect)$ could be Nash equilibriums. From Lemma 4.2, we can see that at any given price, manufacturer could secure a higher profit y delegating the product recovery function to the retailer. For both manufacturers, indirect colletion mode is a dominant strategy. Therefore, when both manufacturers choose indirect collection mode, no one has the inclination to deviate from the equilibrium state. An interesting observation from above proposition is that manufacturer may also simultaneously adopt direct collection when the effective ratio of collection is high and the intensity of competition is high. Direct collection leads to a higher retail price. Charging a higher price has two effects on manufacturer's profit. First, a higher price leads to lower demand, which is a negative effect on manufacturer's profit. On the other hand, a higher price also induces the competitor to price higher, which is a positive effect. When the intensity of competition is high, the second force dominates. Furthermore, the effective ratio of collection sets a threshold for the transformation. Only when the effective ratio is large enough could there exists a threshold of competition intensity above which direct collection mode stands out in the equilibrium state. The threshold $\beta_1(n)$ is decreasing with the improvement of effective ratio of collection. The available range is depicted at the upper right corner of Figure 4.2. The horizontal axis is the effective ratio of collection n while the vertical axis is the intensity of competition β.

In this chapter, the effective ratio of collection n and intensity of competition (product substitutability) β are exogenously given by the competing environment. There're other research investigating the interaction of remanufacturing and the product design. In Esenduran and Kemahlıoğlu-Ziya (2015), manufacturer could make integrated strategies consisting of product differentiation and collection decision.

In sum, manufacturers choose the same product recovery strategies and no one has the inclination to deviate unilaterally. A common peril of Nash Equilibrium is the possibility of prisoners' dilemma. Although indirect collection by retailer is advantageous in bilateral monopoly market, it is possible for competing supply chains to get trapped in a simultaneous move.

Proposition 4.4. *When manufacturer is the second-stage Stackelberg-leader,*

(1) If (Direct, Direct) is the equilibrium, it is Pareto efficient;

(2) (Indirect, Indirect) is Pareto efficient when $0 < \beta < \beta_2(n)$.

When (*Direct, Direct*) is Nash equilibrium, not a single manufacturer could deviate to obtain a higher profit without lowering the other manufacturer's profit. But it is possible that $(Indirect, Indirect)$ is Pareto dominated by (*Direct, Direct*) when the intensity of competition is high. Figure 4.2 depicts the distribution areas of equilibrium when manufacturers function as the Stackelberg leader. In the whole area, $(Indirect, Indirect)$ is the Nash Equilibrium. Only at the top right corner $(Direct, Direct)$ is Nash equilibrium. But in the intermediate range, (*Indirect, Indirect*) arises as prisoners' dilemma for manufacturers.

The reason of the prisoners' dilemma is rooted at the interaction of inter- and inter-channel effect of product recovery. When manufacturer is the Stackelberg leader, the issue of double marginalization makes him prefer to insert retailer into the reverse channel. Retailers are more effective to collect the used products and contribute to lower average production costs. Therefore, supply chain with indirect collection mode

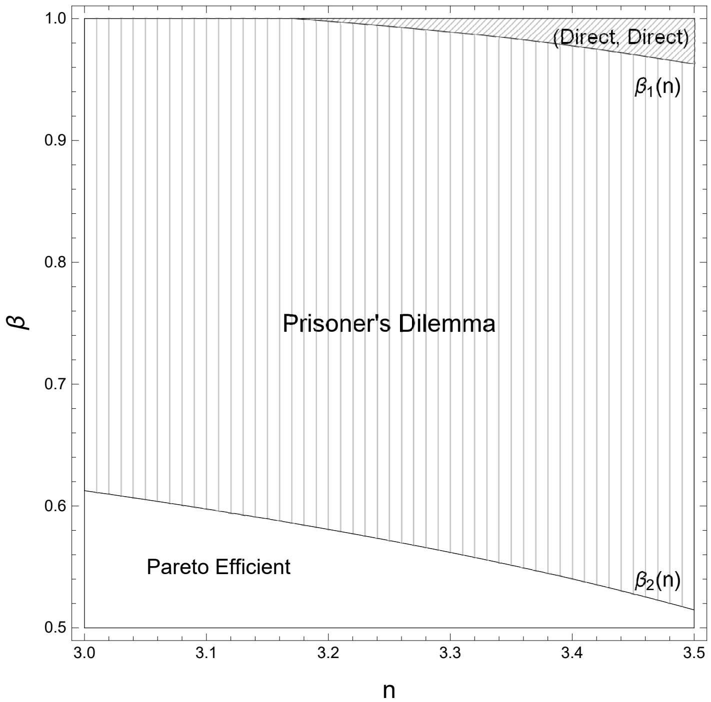


Figure 4.2: Manufacturer leader: first-stage equilibrium

charges a lower price, intensifying the competition. When competition intensity between channels is large, the aggravated competition brings suboptimal profit to the manufacturers, resulting in "lose-lose" situation.

In addition to the profitability of the firms, product recovery strategies influence the social welfare. According to Singh and Vives (1984), we know that the customers with utility function given by $U(q_1, q_2) = \frac{\mu(q_1+q_2)}{1-\beta} - \frac{q_1^2+q_2^2+2\beta q_1 q_2}{2(1-\beta^2)}$ could lead to the demand functions in our model. From the equilibrium results, we can calculate the social welfare of the two equlibria when manufacturer is the Stackelberg - leader. By comparing the social welfare from $(Direct, Direct)$ and $(Indirect, Indirect)$, we find that the social welfare from $(Indirect, Indirect)$ is always higher than $(Direct, Direct)$. Adopting indirect recovery mode, the competing supply chains charge lower prices which induce higher demands and therefore the higher social welfare.

4.4.2 Stackelberg - Retailer as the leader

When retailers act as the Stackelberg leader, retailers move before manufacturer when determining the operations decisions. Given the first-stage strategy combination, retailers determine the operations decisions anticipating the manufacturer's best response.

Second-stage Analysis

We firstly analyze the unilateral response functions of direct collection supply chain and indirect collection supply chain given the competitor's retail price.

When the supply chain adopts direct collection mode, given the retail margin, manufacturers determine the wholesale margins and collection rates by maximizing (4.2). The first-order condition gives the response functions: $\tilde{M}_i^D(\tilde{m}_i^D, p_{3-i}) = \frac{2B(\mu-(c+\tilde{m}_i^D)+\beta p_{3-i})}{4B-\Delta^2}$ and $\tilde{\tau}_i^D(\tilde{m}_i^D, p_{3-i}) = \frac{\Delta(\mu-(c+\tilde{m}_i^D)+\beta p_{3-i})}{4B-\Delta^2}$. Anticipating the response of manufacturer, retailer determines the retail margin by maximizing (4.1). The response functions are represented as the function of competing supply chain's price:

$$\tilde{m}_i^D(p_{3-i}) = \frac{\mu - c + \beta p_{3-i}}{2}. \tag{4.9}$$

Substituting into the wholesale margin and collection rate, we can obtain that:

$$\tilde{M}_i^D(p_{3-i}) = \frac{B(\mu - c + \beta p_{3-i})}{4B - \Delta^2} \text{ and } \tilde{\tau}_i^D(p_{3-i}) = \frac{B(\mu - c + \beta p_{3-i})}{2(4B - \Delta^2)}. \tag{4.10}$$

The resulting price is given by $\tilde{p}_i^D(p_{3-i}) = \frac{Bc+(3B-\Delta^2)(\mu-\beta p_{3-i})}{4B-\Delta^2}$. The expected profit of manufacturer is $\tilde{\Pi}_i^D(p_{3-i}) = \frac{B(\mu-c+\beta p_{3-i})^2}{4(4B-\Delta^2)}$.

When the supply chain uses indirect collection mode, manufacturer maximizes (4.4) to determine the wholesale margin and buyback payment. For a given buyback payment $\tilde{b}_i$, manufacturer's profit is concave in wholesale margin. Hence, the optimal wholesale margin is given by the first-order condition $\tilde{M}_i^I(\tilde{m}_i^I, \tilde{\tau}_i^I, p_{3-i}) = \frac{1}{2}(\mu - (c - (\Delta - b_i)\tilde{\tau}_i^I + \tilde{m}_i^I) + \beta p_{3-i})$. The manufacturer's profit is given by $\frac{1}{4}(\mu - (c - (\Delta - b_i)\tilde{\tau}_i^I + \tilde{m}_i^I) + \beta p_{3-i})^2$, which is decreasing with b_i. Hence, we set the optimal buyback payment equal to zero, i.e., $\tilde{b}_i^I = 0$. Substituting $\tilde{M}_i^I$ and $\tilde{b}_i = 0$ into (4.3), retailer determines the optimal retail margin and collection rate as follows:

$$\tilde{m}_i^I(p_{3-i}) = \frac{4B(\mu - c + \beta p_{3-i})}{8B - \Delta^2} \text{ and } \tilde{\tau}_i^I(p_{3-i}) = \frac{\Delta(\mu - c + \beta p_{3-i})}{8B - \Delta^2}. \tag{4.11}$$

The wholesale margin is given by

$$\tilde{M}_i^I(p_{3-i}) = \frac{2B(\mu - c + \beta p_{3-i})}{8B - \Delta^2}. \tag{4.12}$$

From the above results, we can calculate the resulting retail price as $\tilde{p}_i^I(p_{3-i}) = \frac{2Bc + (6B - \Delta^2)(\mu + \beta p_{3-i})}{8B - \Delta^2}$ and the expected profit of manufacturer is $\frac{4B^2(\mu - c + \beta p_{3-i})^2}{(8B - \Delta^2)^2}$. The second-stage equilibrium results can be calculated by the interaction of corresponding response functions.

The equilibrium results of $(Direct, Direct)$ scenario could be derived from the intersection of the response functions of supply chain adopting direct collection mode (retailer (4.9) and manufacture (4.10)). The intersection of retailer's response function (4.11) and manufacturer's response function (4.12) gives the equilibrium result of $(Indirect, Indirect)$ scenario. Similarly, the intersection of the response function of a supply chain adopting direct collection mode and the response function of a supply chain adopting indirect collection mode gives the equilibrium results of $(Direct, Indirect)$ scenario. The equilibrium results could be found in the appendix.

The response functions enable us to look into the feature of different product recovery strategy before deriving the first-stage Nash equilibrium. The following lemma compares the retail prices and collection rates of two product recovery strategies given the competitor's retail price.

Lemma 4.5. *When retailer is the Stackelberg leader, given the retail price of competing supply chain, supply chain adopting direct collection mode collects more,* $\tilde{\tau}_i^D(p_{3-i}) > \tilde{\tau}_i^I(p_{3-i})$*, and charges lower price,* $\tilde{p}_i^D(p_{3-i}) < \tilde{p}_i^I(p_{3-i})$*.*

When retailer is the Stackelberg leader, he is cursed by the double marginalization. That is, the cost saving from remanufacturing can only be partially reflected on the final price when retailer assumes the duty of product collection. Therefore, retailer has less incentive to collect used products, making the collection rate under indirect collection mode lower than the collection rate under direct collection mode. The average production cost with direct collection mode is also lower. The lower production cost under direct collection mode enables the supply chain to charge a lower price. Therefore, the direct collection is also beneficial to consumers because they can enjoy lower prices. The following lemma gives the order of manufacturers' margins and profits under different product recovery strategies.

Lemma 4.6. *When retailer is the Stackelberg leader, given the retail price of competing supply chain, manufacturer under direct collection mode charges a higher margin,* $\tilde{M}_i^D(p_{3-i}) > \tilde{M}_i^I(p_{3-i})$*, and obtains a higher profit,* $\tilde{\Pi}_i^D(p_{3-i}) > \tilde{\Pi}_i^I(p_{3-i})$*.*

When manufacturer is not the channel leader, he could charge a larger margin and obtain a higher profit under direct collection mode given the competing supply chain's price. He is more effective to collect the used products because the cost saving can be directly reflected on the retail price. The average production cost is lower. Hence, manufacturer could charge higher margin. With direct collection mode, the price is also lower. Therefore, a larger demand and a higher margin make the direct collection more profitable for the manufacturer, although the manufacture has to afford the cost to invest in the product recovery.

First-stage Analysis

Based on the second-period equilibrium results, we're ready to look at the equilibrium of the first stage. When the retailer is the Stackelberg-leader, the property of product recovery strategy for the manufacturer is changed. The following proposition presents the first-stage equilibrium.

Proposition 4.7. *When retailer is the second-stage Stackelberg-leader,*
(1) (Direct, Direct) is always Nash Equilibrium;
(2) (Indirect, Indirect) is Nash Equilibrium when $n > 2\sqrt{3}$ *and* $\beta > \beta_3(n)$.

Similar to the case when manufacturer is the Stackelberg-leader, both (*Direct, Direct*) and (*Indirect, Indiret*) could be Nash equilibrium but the specific conditions are different. From Lemma 4.6, we know that direct collection is basically a more preferable strategy for the manufacturer given the competitor's price. Therefore, $(Direct, Direct)$ is always a Nash equilibrium and no one has the inclination to unilaterally deviate.

On the other hand, manufacturers may also simultaneously adopt indirect collection mode. From Lemma 4.6, we know that the retail price under indirect collection mode is higher. As we argued in the former section, the effect of low-price strategy is two-fold. A higher price leads to a lower demand, which is a negative effect on manufacturer's profit. In the meanwhile, a higher price also has a positive effect on manufacturer's profit because a higher price induces the competitors to charge a higher prices as well. When the intensity of competition is large, the positive effect dominates. Figure 4.3 depicts the distribution areas of equilibrium when retailer is the Stackelberg leader. The horizontal axis is the effective ratio of collection n while the vertical axis is the intensity of competition β. In the whole area, (*Direct, Direct*) is the Nash equilibrium. The range for the (*Indirect, Indirect*) becoming a equilibrium is shown in the upper right corner in Figure 4.3, where the effective ratio of large is large and the competition intensity is high. The following proposition shows that the Pareto efficiency of the Nash equilibriums.

Proposition 4.8. *When retailer is the second-stage Stackelberg-leader,*

(1) If (Indirect, Indirect) is the equilibrium, it is Pareto efficient;

(2) (Direct, Direct) is Pareto efficient when $n > 2\sqrt{3}$ *and* $\beta < \beta_4(n)$.

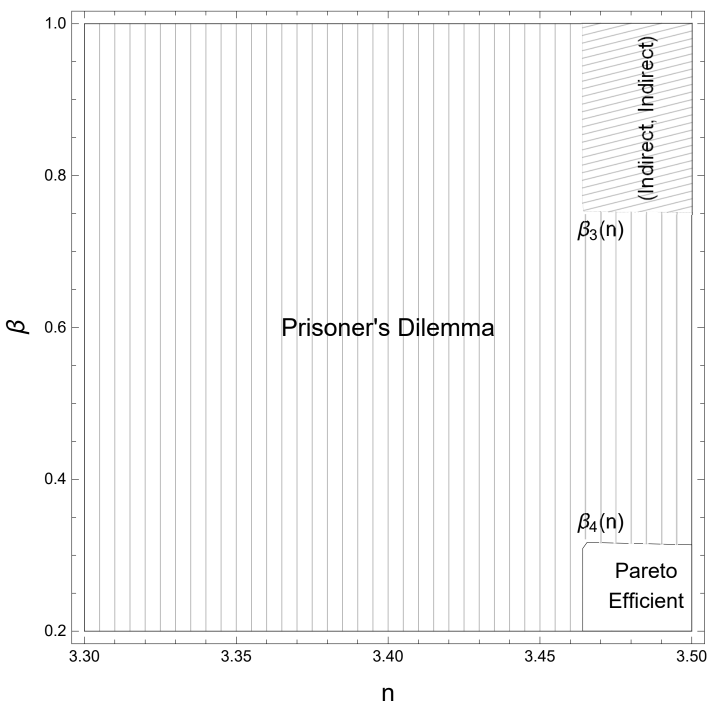


Figure 4.3: Retailer leader: first-stage equilibrium

(*Indirect, Indirect*) is always Pareto efficient but prisoners' dilemma arises when both manufacturers choose direct collection mode. (*Direct, Direct*) is Pareto efficient only when the competition intensity is low and the effective ratio of collection is relatively large. When retailer is the Stackelberg leader, manufacturer has more incentive to collect the used products directly. The supply chain with direct collection mode charges a lower price due to the lower average production cost from the more effective collection. A low prices represents more intensified competition between the supply chains. The low price strategy drags the manufacturers into the prisoners' dilemma when the effective ratio of collection is not high enough and (or) the competition intensity is relatively high.

Likewise, we also compare the social welfare under (*Direct, Direct*) and (*Indirect, Indirect*) and find that the social welfare under (*Direct, Direct*) equilibrium is always higher. The supply chain charges lower price when the manufacturer adopts *Direct* recovery strategy. This is because that the manufacturer is more effective to increase the collection rate and the supply chain is able to produce at a more cost-effective level. Lower prices bring larger demands and therefore higher social welfare.

4.4.3 Vertical Nash

In this subsection, we analyze the situation when the relationship of manufacturer and retailer in either supply chain is vertical Nash. We also analyze the problem by backward induction.

Second-stage Analysis

Given the strategy combination formed in the first stage, manufacturers and retailers determine the operations decisions including pricing and collection rate in the second stage. When manufacturer and retailer within a supply chain are engaged in vertical Nash, they move simultaneously to determine the operations decisions. Therefore, the intersection of the response functions of manufacturers and retailers from two supply chains gives the second-stage equilibrium. The following proposition characterizes the second-stage equilibrium result.

Proposition 4.9. *When the second-stage pricing game is vertical Nash, the supply chain decisions are the same for different first-stage strategy combinations. The equilibrium results are characterized as follows:*

(1) The profit margins of the retailer and manufacturer are given by

$$\hat{m}_i = \hat{M}_i = \frac{2B(\mu - (1-\beta)c)}{(6-4\beta)B - (1-\beta)\Delta^2}.$$

(2) The collection rate is given by

$$\hat{\tau}_i = \frac{\Delta(\mu - (1-\beta)c)}{(6-4\beta)B - (1-\beta)\Delta^2}.$$

(3) The profits of retailers and manufacturers are given by

$$\hat{\pi}_i^D = \hat{\Pi}_i^I = \frac{4B^2(\mu - (1-\beta)c)^2}{((6-4\beta)B - (1-\beta)\Delta^2)^2},$$

$$\hat{\pi}_i^I = \hat{\Pi}_i^D = \frac{B(4B - \Delta^2)(\mu - (1-\beta)c)^2}{((6-4\beta)B - (1-\beta)\Delta^2)^2}.$$

From the above proposition, we can see that when there's no pricing leadership in supply chain, the issue of double marginalization does not bother the firms. All firms charge equal margins no matter which product recovery strategy they adopt. The collection rates in both direct and indirect collection supply chain are also the same. The profitability of a firm depends on whether he assumes the duty of product collection. The firm's profit is lower if he is responsible to collect the used products. Because the firm has to afford the related investment cost if he collects the used products. The closeness to consumer has no effect on profit. The manufacturer in direct collection supply chain gains the same profit as the retailer in indirect collection supply chain.

First-stage Analysis

When the second-stage is vertical Nash, the manufacturer's profit with either collection mode is invariant with the competing supply chain's product recovery strategy. The effect of competition on choosing collection mode is nullified. This particular feature changes the equilibrium in the first stage as shown in the following proposition.

Proposition 4.10. *When the second-stage pricing game is vertical Nash, (Indirect, Indirect) is always the unique Nash equilibrium and it is also Pareto efficient.*

When the manufacturer and retailer decide simultaneously at the second stage, the indirect collection mode is basically a more preferable strategy for the manufacturers. Therefore, $(Indirect, Indirect)$ is always the Nash equilibrium. Without the influence of double marginalization, both manufacture and retailer are equally effective to collect the used products. Since the collection rates are the same, the average production costs are the same for direct and indirect collection modes. The prices of the product under different strategy combinations are also the same. Neither manufacturer has incentive to deviate.

From the perspective of social welfare, we find that the equilibrium $(Indirect, Indirect)$ has the same social welfare as the other strategy combinations. This is because that when manufacturers and retailer are engaged in vertical Nash, the retail prices are the same across different strategy combinations.

4.5 Conclusion

This paper investigates the inter- and intra- channel implications of the product recovery. We model the problem in two competing supply chains, each consisting of one manufacturer and one retailer. There're three types of power structure in the supply chain, i.e., Stackelberg - manufacturer as the leader, Stackelberg - retailer as the leader and vertical Nash. Manufacturers have incorporated cost-effective remanufacturing system so that the products could be produced by raw material and collected used parts. Manufacturesrs have two options regarding to the product recovery strategy, collecting used products for remanufacturing by itself (that is, direct recovery) and assigning the task of product recovery to its retailer (indirect recovery). The problem is formulated as a two-stage game. Both manufacturers determine the product recovery strategy simultaneously in the first stage. In the second stage, firms determine the operations decisions in accordance with the strategy combinations.

Our analysis indicates when the manufacturers and the retailers engage in a vertical Nash game, indirect recovery is the unique equilibrium and is also Pareto efficient. However, when there's leadership in the supply chain, multiple equilibria occur when the competition intensity and effective ratio of collection are high, thus either direct recovery or indirect recovery may be chosen. Specifically, the firm with channel power has less incentive to increase the collection rate. A higher collection rate reduces the average production cost and enables the supply chain to charge a lower retail price. At equilibrium, manufacturers would adopt the product recovery strategy which can achieve a lower price. However, manufacturers may be trapped into prisoners' dilemma due to the intensified competition. On the other hand, the product recovery strategy which leads to a higher retail price can turn to be a Pareto efficient Nash equilibrium when the competition intensity and effective ratio of collection are high.

With a better understanding of the product recovery strategy, the closed-loop supply chain management could provide more guidance to the practitioners. Competition intensity, product collection efficiency and supply chain power structure influence which product recovery strategy the manufacturer would like to adopt. Therefore, we observe that manufacturers like Xerox, Lenovo, Epson etc. collect the used products for remanufacturing by themselves. On the other hand, manufacturers like Caterpillar and HP collaborate with the retailer to collect the used products. The managerial implication from the research is that pursuing the product recovery strategy which brings a higher collection rate and a lower retail price may make the manufacturers be trapped in the prisoners' dilemma.

Our model assumes that manufacturers and retailers have the same recovery costs B. We tried but find analytically difficult to extend our model to consider the differentiated recovery costs between manufacturers and retailers. We conjecture that our results qualitatively hold but the unbalanced recovery costs will influence the thresholds. For example, when the manufacturers' recovery cost is lower (or higher), *Direct* (or *Indirect*) recovery is more likely to be the equilibrium strategy.

We assume that the firms have complete information in the game. In reality, we can see that in many cases retailers may be more informative about the market. By incorporating the asymmetric information, we can get more insights about the product recovery. The other direction for future research is the differentiated pricing for the products and remanufactured products.

Chapter 5

Conclusions

With the intensified competition and technological advancement, new operations problems arise. The competition background endows the operations problems with new implications. We study three emerging operations problems including learning-by-doing effect, trade credit, and product recovery in this thesis.

The first study investigates the coopetition effect of learning-by-doing. The learning-by-doing effect intensifies the competition when the cooperation between the OEMs is eliminated. The learning effect could decrease the OEMs' total profit compared to the case with no learning. When OEMs outsource to a common CM, we find that OEMs have incentive to enhance the pooled cost reduction. The OEMs' total profits under this situation are always higher than the case with no learning. Moreover, When the competition intensity and the learning speed are relatively low, OEMs' total profits are increasing with the competition intensity, reflecting the dominant role of cooperation effect. The dominant cooperation effect is robust when considering the pricing power of CM and other pricing strategies including uniform pricing and myopic pricing. We also find that when the OEMs are differentiated in the market sizes, the OEM with a much larger market size may prefer to outsource to a separate CM instead.

The second study examines the usage and effectiveness of trade credit in the joint vertical and horizontal relationship. The trade credit changes the competing behaviors of the supply chain members. First of all, the bail out effect is identified. We find that the supplier may provide a lower adjusted wholesale price to the financially distressed retailer when it is competing with a retailer who has sufficient capital, compared to the case where two financially distressed retailers compete with each other. Secondly, the predatory effect is observed in the case where two retailers have different financial statuses. However, the predatory is bidirectional here. The retailer with sufficient capital may sell more to predate the financially distressed retailer when the variance of demand shock is moderate while the financially distressed retailer also sells more when the variance of demand shock is large. Compared to the case where both retailers are financially distressed, the supplier may prefer the case where retailers have unbalanced financial statuses when the variance of demand shock is moderate. The supplier may prefer the case where both retailers are financially distressed when the variance of demand shock is relatively large. Better financial status would bring a higher profit for the retailer no matter what the financial status of its competitor is. However, the improvement of its competitor's financial status is favorable only when the variance of demand shock is relatively high.

The third study investigates product recovery strategies in competing supply chains. Either manufacturer can choose between two product recovery strategies, collecting used products for remanufacturing by itself (that is, direct recovery) and assigning the task of product recovery to its retailer (indirect recovery). The manufacturer and retailer in either supply chain are engaged in three types of game sequence: Stackelberg-manufacturer as the leader, Stackelberg-retailer as the leader, and vertical Nash. Our analysis indicates when the manufacturers and the retailers engage in a vertical Nash game, indirect recovery is the unique equilibrium and is Pareto efficient.

However, when the parties engage in a Stackelberg leader-follower game, multiple equilibrium occur when the competition intensity is high, and thus either direct recovery or indirect recovery may be chosen. The channel power determines either direct recovery or indirect recovery as the low-price strategy. Specifically, the firm with channel power has less incentive to increase the collection rate. A higher collection rate reduces the average production cost and enables the supply chain to charge a lower retail price. At equilibrium, manufacturers would adopt the product recovery strategy which can achieve a lower price. Manufacturers may be trapped into prisoners' dilemma for choosing the low-price strategy.

Appendix A

Proofs in Chapter 2

Proof of Proposition 2.1

We first derive the equilibrium results for the case of no learning and separate learning and then compare the prices and profits.

Equilibrium of No Learning When the production does not exhibit learning effect, the decision in either period won't interact. The problem is reduced to one shot decision. OEMs simultaneously determine the price to maximize the profit function:

$$\pi_{it}^{B} = q_{it}(p_{it} - c - k)$$

The profit function is concave in the price p_{it}, the first-order condition gives the best response functions for both OEMs $p_{it}(p_{jt}) = \frac{a+(1+\theta)(c+k)+\theta p_{jt}}{2(1+\theta)}$. The intersection gives the equilibrium prices $p_{it}^{B*} = \frac{a+(1+\theta)(c+k)}{2+\theta}$. Hence OEMs' equilibrium profit in either period is $\pi_{it}^{B*} = \frac{(1+\theta)(a-c-k)^2}{(2+\theta)^2}$.

Equilibrium of Joint Learning We solve this two-period problem by backward induction. At the beginning of the second period, given the first-period production quantities q_{i1} and q_{j1}, OEMs simultaneously decide the price to maximize the second-period profit,

$$\pi_{i2}(q_{i2}, q_{j2}) = (p_{i2} - (c - \lambda(q_{i1} + q_{j1}) + k))q_{i2}$$

The first order condition gives the response function for the two OEMs: $p_{i2} = \frac{a+(1+\theta)(c-\lambda(q_{i1}+q_{j1})+k)+\theta p_{j2}}{2(1+\theta)}$. The second period equilibrium price is derived by the intersection of the OEMs' response functions:

$$p^*_{i2}(p_{i1}, p_{j1}) = \frac{a + (1+\theta)(c - \lambda(2a - p_{i1} + p_{j1}) + k)}{2+\theta}$$

The equilibrium profit of the second period is given as a function of the first-period price $\pi^*_{i2}(p_{i1}, p_{j1}) = \frac{(1+\theta)(a-(c-\lambda(2a-p_{i1}+p_{j1})+k))^2}{(2+\theta)^2}$. In the first period, the OEMs determine the first-period price to maximize the total expected profit considering the cost saving in the later period.

$$\pi_i(p_{i1}, p_{j1}) = (p_{i1} - (c+k))q_{i1} + \pi^*_{i2}(p_{i1}, p_{j1})$$

The total profit is concave in p_{i1}. Again, we can obtain the best response of either OEM from the first order condition and the first period equilibrium price is computed by the intersection of the response functions.

$$p^{J*}_{i1} = \frac{((2+\theta)^2 - 2\lambda(1+\theta) - 4\lambda^2(1+\theta))a + (1+\theta)((2+\theta)^2 + 2\lambda)(c+k)}{(2+\theta)^3 - 4\lambda^2(1+\theta)}$$

The equilibrium production quantity is then computed as $q^{J*}_{i1} = \frac{(1+\theta)((2+\theta)^2+2\lambda)(a-c-k)}{(2+\theta)^3-4\lambda^2(1+\theta)}$. Substituting into the profit function, we can calculate the total profit at equilibrium as

$$\pi_i^{J*} = \frac{2(1+\theta)(a-c-k)^2}{\left((2+\theta)^3 - 4(1+\theta)\lambda^2\right)^2} \cdot (-4(1+\theta)\lambda^3 + 2(1+\theta)\left(\theta(2+\theta)^2 - 1\right)\lambda^2$$
$$+ ((5+2\theta)\theta + 4)(2+\theta)^2\lambda + (2+\theta)^4)$$

The second-period price is given by $p_{i2}^{J*} = \frac{a}{2+\theta} + \frac{1+\theta}{2+\theta}\left(c + k - \frac{2(1+\theta)\lambda\left((2+\theta)^2+2\lambda\right)(a-c-k)}{(2+\theta)^3-4(1+\theta)\lambda^2}\right)$.

Derivation of Assumption 1

(a) To guarantee the non-negativity of the demand, θ and λ needs to satisfy the inequality $(2+\theta)^3 - 4(1+\theta)\lambda^2 > 0$ that is $\lambda < \frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}$.

(b) The second period unit production cost is given by $c - \lambda(q_{11} + q_{21}) > 0$. Therefore, the first-period cost can't be too small, $c > \frac{4\lambda^2(1+\theta)+2\lambda(1+\theta)(2+\theta)^2}{(2+\theta)^3+2\lambda(1+\theta)(2+\theta)^2}(a-k)$. Since $4\lambda^2(1+\theta) < (2+\theta)^3$, $\frac{4\lambda^2(1+\theta)+2\lambda(1+\theta)(2+\theta)^2}{(2+\theta)^3+2\lambda(1+\theta)(2+\theta)^2}$ is a fraction between 0 and 1 in the available range of θ and λ required in (a).

Price and Profit Comparisons Based on the equilibrium results, we can proceed to analysis the differences between prices and profits.

First period price

$$p_{i1}^{J*} - p_{it}^{B*} = -\frac{2(1+\theta)\lambda(2(1+\theta)\lambda + \theta + 2)(a-c-k)}{(2+\theta)^4 - 4(1+\theta)(2+\theta)\lambda^2} < 0$$

Second period price

$$p_{i2}^{J*} - p_{it}^{B*} = -\frac{2(\theta+1)^2\lambda\left((\theta+2)^2 + 2\lambda\right)(a-c-k)}{(\theta+2)^4 - 4(\theta+1)(\theta+2)\lambda^2} < 0$$

We can also check that under joint learning, the first period price is always higher than the second period price.

$$p_{i1}^{J*} - p_{i2}^{J*} = \frac{2(\theta+1)(\theta(\theta+3)+1)\lambda(a-c-k)}{(\theta+2)^3 - 4(\theta+1)\lambda^2} > 0$$

First period profit

The first period profit with joint learning is obviously lower than the case without learning, since the costs are the same while the joint learning case charges lower price.

Second period profit

$$\pi_{i2}^{J*} - \pi_{it}^{B*}$$
$$= \frac{8\lambda(1+\theta)(a-c-k)^2}{(2+\theta)^2((2+\theta)^3 - 4\lambda^2(1+\theta))^2}\left(2(2+\theta)^3 + (2+\theta)^5 + \lambda(1+\theta)(2+\theta)^4 + 4\lambda^3(1+\theta)\right) > 0$$

Total profit

$$\pi_i^{J*} - \pi_i^{B*}$$
$$= \frac{2\lambda(1+\theta)(a-c-k)^2}{(2+\theta)^2((2+\theta)^3 - 4(1+\theta)\lambda^2)^2}$$
$$\times(-16(\theta+1)^2\lambda^3 - 4(\theta+1)(\theta+2)^2\lambda^2 + 2(\theta+1)(\theta(\theta(\theta+4)+8)+7)(\theta+2)^2\lambda$$
$$+ (\theta(2\theta+5)+4)(\theta+2)^4)$$

Excluding the terms which are always positive, the key term is $M(\lambda) = -16(\theta+1)^2\lambda^3 - 4(\theta+1)(\theta+2)^2\lambda^2 + 2(\theta+1)(\theta(\theta(\theta+4)+8)+7)(\theta+2)^2\lambda + (\theta(2\theta+5)+4)(\theta+2)^4$. We can show that the term is concave in λ. The values of the term at $\lambda = 0$ and $\lambda = \frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}$ are all positive. Therefore, we can conclude that the term is always positive. The total profit with joint learning is larger than the total profit without learning.

Proof of Proposition 2.2

We also derive the equilibrium result by backward induction. In the second period, OEMs, knowing the first period quantity q_{i1}, determine the second-period selling price to maximize the profit, with the new production cost being $c - \lambda q_{i1}^S$.

$$\max_{p_{i2}^S} \ (p_{i2}^S - (c - \lambda q_{i1}^S + k))q_{i2}^S$$

The profit function is concave in the price, hence the first order condition gives the response function of the OEMs:

$$p_{i2}^S(p_{j2}^S) = \frac{a + (1+\theta)(c - \lambda q_{i1}^S + k) + \theta p_{j2}^S}{2(1+\theta)}$$

The intersection of the response function gives the second-period equilibrium price. The price is given as

$$p_{i2}^S = \frac{(2+3\theta)(a + (1+\theta)(c+k)) - \lambda(1+\theta)(2(1+\theta)q_{i1}^S + \theta q_{j1}^S)}{(2+\theta)(2+3\theta)}$$

Substituting into the profit function, the equilibrium profit of the OEMs in the second-period is given by

$$\pi_{i2}^S = \frac{(1+\theta)\left((2+3\theta)(a-c-k) + \lambda(2+\theta(4+\theta))q_{i1}^S - \lambda\theta(1+\theta)q_{j1}^S\right)^2}{(4+8\theta+3\theta^2)^2}$$

With the first period demand given by the corresponding prices: $q_{i1}^S = a - p_{i1}^S + \theta(p_{j1}^S - p_{i1}^S)$. Move to the first period, we can derive the first period price of OEMs by maximizing the total profit

$$\max_{p_{i1}^S} \ (p_{i1}^S - c - k)q_{i1}^S + \pi_{i2}^S$$

The concavity of the total profit requires $0 < \lambda < \frac{4+8\theta+3\theta^2}{2+6\theta+6\theta^2+2\theta^3}$ for any $\theta > 0$. The equilibrium first-period price can then be obtained from the intersection of the best response functions which could be derived by the first order condition:

$$p_{i1}^{S*} = \frac{4\lambda(1+\theta)^4(a-c-k) + 4a\lambda^2(1+\theta)^4 - (2+\theta)^2(2+3\theta)(a+(1+\theta)(c+k))}{4\lambda^2(1+\theta)^4 - (2+\theta)^3(2+3\theta)}$$

The first period quantity is $q_{i1}^{S*} = a - p_{i1}^{S*}$, and we can obtain the second period price,

$$p_{i2}^{S*} = \frac{\lambda(1+\theta)^4(a-c-k) + 4a\lambda^2(1+\theta)^4 - (2+\theta)^2(2+3\theta)(a+(1+\theta)(c+k))}{4\lambda^2(1+\theta)^4 - (2+\theta)^3(2+3\theta)}$$

The second period profit is given by

$$\pi_{i2}^{S*} = \frac{(\theta+1)(\theta+2)^2(3\theta+2)^2(\theta\lambda+\theta+\lambda+2)^2(a-c-k)^2}{16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2}$$

The total profit could be given by $\pi_i^{S*} = q_{i1}^{S*}(p_{i1}^{S*} - c - k) + \pi_{i2}^{S*}$. We can also check that the non-negativity condition for the demand is satisfied under the concavity condition.

Price and Profit Comparisons

First period price

$$p_{i1}^{S*} - p_{it}^{B*} = \frac{4(\theta+1)^4\lambda(\theta\lambda+\theta+\lambda+2)(a-c-k)}{4(\theta+1)^4(\theta+2)\lambda^2 - (\theta+2)^4(3\theta+2)}$$

We can find that $4(\theta+1)^4(\theta+2)\lambda^2 - (\theta+2)^4(3\theta+2)$ is negative in the available range of λ and θ. Therefore, the first period price under separate learning is lower than the price in the base case without learning.

Second period price

$$p_{i2}^{S*} - p_{it}^{B*} = \frac{\lambda\left(4(\theta+1)^5\lambda + (\theta+2)^2(3\theta+2)(\theta+1)^2\right)(a-c-k)}{(2+\theta)\left(4(\theta+1)^4\lambda^2 - (\theta+2)^3(3\theta+2)\right)} < 0$$

We can also find that the first period price under separate learning is lower than the first period price.

$$p_{i1}^{S*} - p_{i2}^{S*} = \frac{\theta^2(\theta+1)^2\lambda(a-c-k)}{4(\theta+1)^4\lambda^2 - (\theta+2)^3(3\theta+2)} < 0$$

First period profit

The first period profit is always lower than the base case without learning because the firms charge lower prices while produce at the same cost.

Second period profit

$$\begin{aligned}
&\pi_{i2}^{S*} - \pi_{it}^{B*} \\
&= \frac{\lambda(a-c-k)^2(1+\theta)^2}{(2+\theta)^2} \\
&\times \left(\frac{-16(\theta+1)^7\lambda^3 + (\theta+2)^3(3\theta+2)(\theta(11\theta+24)+12)(\theta+1)\lambda + 2(\theta+2)^5(3\theta+2)^2}{16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2}\right)
\end{aligned}$$

We can check that $16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2$ is positive. Since $-16(\theta+1)^7\lambda^3 + (\theta+2)^3(3\theta+2)(\theta(11\theta+24)+12)(\theta+1)\lambda + 2(\theta+2)^5(3\theta+2)^2$ is concave in λ, and it is positive when $\lambda = 0$ and $\lambda = \frac{4+8\theta+3\theta^2}{2+6\theta+6\theta^2+2\theta^3}$. The term is positive in the available range of λ. Therefore, the second period profit under separate learning is higher than the profit in the case without learning.

Total profit

$$
\begin{aligned}
&\pi_i^{S*} - \pi_i^{B*} \\
&= \lambda(a-c-k)^2(1+\theta)^2 \\
&\times (\frac{-16(\theta+2)^2(\theta+1)^6\lambda^2 - (\theta+2)^2(\theta(\theta(\theta(\theta(28\theta+103)+92)-48)-96)-32)(\theta+1)\lambda}{(2+\theta)^2\left(16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2\right)} \\
&+ \frac{-2(\theta+2)^4(3\theta+2)\left(\theta\left(2\theta^2+\theta-6\right)-4\right) - 32(\theta+1)^7\lambda^3}{(2+\theta)^2\left(16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2\right)})
\end{aligned}
$$

We can check that the denominator $(2+\theta)^2(16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2+(\theta+2)^6(3\theta+2)^2)$ is positive. The numerator, $-2(\theta+2)^4(3\theta+2)(\theta\left(2\theta^2+\theta-6\right)-4) - 32(\theta+1)^7\lambda^3 - 16(\theta+2)^2(\theta+1)^6\lambda^2 - (\theta+2)^2(\theta(\theta(\theta(\theta(28\theta+103)+92)-48)-96)-32)(\theta+1)\lambda$, is concave in λ. When $\lambda = \frac{3\theta^2+8\theta+4}{2\theta^3+6\theta^2+6\theta+2}$, the numerator is negative. When $\lambda = 0$, the numerator equals $-2(\theta+2)^4(3\theta+2)\left(\theta\left(2\theta^2+\theta-6\right)-4\right)$. If this is negative, then the numerator is negative in the whole area. We can check that when $\theta > 1.7938$, $-2(\theta+2)^4(3\theta+2)\left(\theta\left(2\theta^2+\theta-6\right)-4\right)$ is negative. On the other hand, when $0 < \theta < 1.7938$, $-2(\theta+2)^4(3\theta+2)\left(\theta\left(2\theta^2+\theta-6\right)-4\right)$ is positive and we can show the numerator either decreases with λ or increases first then decreases with λ. Therefore, there exists a $\lambda_1(\theta)$ such that when $0 < \lambda < \lambda_1(\theta)$ the total profit with separate learning is higher than the total profit without learning.

Proof of Lemma 2.3

To compare the two-period total profit under separate learning and joint learning, we can get

$$\pi_i^{J*} - \pi_i^{S*}$$
$$= (a-c-k)^2(1+\theta)$$
$$\times \Big(\frac{2(-4(\theta+1)\lambda^3 + 2(\theta+1)\left(\theta(\theta+2)^2-1\right)\lambda^2 + (\theta(2\theta+5)+4)(\theta+2)^2\lambda + (\theta+2)^4)}{\left((\theta+2)^3-4(\theta+1)\lambda^2\right)^2}$$
$$- \frac{-2(\theta+2)^2(3\theta+2)(\theta(2\theta^2+\theta-6)-4)(\theta+1)\lambda - 16(\theta+1)^7\lambda^3}{16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2}$$
$$- \frac{-(\theta(\theta(\theta(\theta(28\theta+151)+316)+320)+160)+32)(\theta+1)^2\lambda^2 + 2(\theta+2)^4(3\theta+2)^2}{16(\theta+1)^8\lambda^4 - 8(\theta+2)^3(3\theta+2)(\theta+1)^4\lambda^2 + (\theta+2)^6(3\theta+2)^2}\Big)$$

We can find that the term is always positive in the joint set of $0 < \lambda < \frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}$ and $0 < \lambda < \frac{3\theta^2+8\theta+4}{2\theta^3+6\theta^2+6\theta+2}$.

Proof of Proposition 2.4

The result of proposition 2.4 could be derived by take the first order derivative of the equilibrium total profit of θ:

$$\frac{d\pi_i^{J*}}{d\theta}$$
$$= \frac{2(\theta+2)(a-c-k)^2}{\left((\theta+2)^3-4(\theta+1)\lambda^2\right)^3}$$
$$\times \left(-2(\theta+1)\left(\theta\left(\theta\left(\theta^2+\theta-4\right)-8\right)-2\right)(\theta+2)\lambda^2 - 8(\theta+1)^3(3\theta+2)\lambda^4\right.$$
$$\left. - 4(\theta+1)(\theta(6\theta(\theta+3)+17)+6)\lambda^3 - (\theta(2\theta(\theta+1)-1)-2)(\theta+2)^3\lambda - \theta(\theta+2)^5\right)$$

We can show that the key term $-2(\theta+1)\left(\theta\left(\theta\left(\theta^2+\theta-4\right)-8\right)-2\right)(\theta+2)\lambda^2 - 8(\theta+1)^3(3\theta+2)\lambda^4 - 4(\theta+1)(\theta(6\theta(\theta+3)+17)+6)\lambda^3 - (\theta(2\theta(\theta+1)-1)-2)(\theta+2)^3\lambda - \theta(\theta+2)^5$ is decreasing with θ. When $\theta = 0$, we can find the key term is positive when $0 < \lambda <$

$\frac{1}{4}\left(\sqrt{17}-1\right)$. When θ is sufficiently large, the key term is negative. Therefore, when $0<\lambda<\frac{1}{4}\left(\sqrt{17}-1\right)$, there exists a threshold $\theta_1(\lambda)$ such than when $0<\theta<\theta_1(\lambda)$ the key term is positive.

Proof of Proposition 2.5

We firstly derive the equilibrium results. The demand in both periods won't fluctuate and the OEMs would determine the uniform price to maximize the total profit of the two periods.

$$\pi_i^U = q_i(p_i - c - k) + q_i(p_i - c + \lambda(q_i + q_j) + k)$$

The total profit is concave in the price. The best response function of either firm is derived by the first order condition. The equilibrium price is derived by the intersection of the best response functions.

$$p_i^{U*} = \frac{(2\theta\lambda + 3\lambda - 2)a - 2(1+\theta)(c+k)}{4 + 2(1-\lambda)\theta - 3\lambda}$$

The demand in both periods is now $q_i^{U*} = \frac{2(1+\theta)(a-c-k)}{4+2(1-\lambda)\theta-3\lambda}$. Substituting into the profit function, we can get the profit in equilibrium as

$$\pi_i^{U*} = \frac{4(1+\theta)(2-\lambda)(a-c-k)^2}{(4+2(1-\lambda)\theta-3\lambda)^2}$$

To ensure the non-negativity of the demand, we assume $0<\lambda<\frac{4+2\theta}{3+2\theta}$. To understand the influence of the competition intensity, we take the first order derivative of the equilibrium profit with θ,

$$\frac{d\pi_i^{U*}}{d\theta} = \frac{4(\lambda-2)(2\theta(\lambda-1)+\lambda)(a-c-k)^2}{(4+2(1-\lambda)\theta-3\lambda)^3}$$

Sine $4+2(1-\lambda)\theta-3\lambda$ is positive, the key term is $4(\lambda-2)(2\theta(\lambda-1)+\lambda)$. When $0<\lambda\leq 1$, the key term is positive if $0<\theta\leq\frac{\lambda}{2-2\lambda}$ and negative is $\theta>\frac{\lambda}{2-2\lambda}$. When $\lambda>1$ and with the constraint of $0<\lambda<\frac{4+2\theta}{3+2\theta}$, the key term is negative.

Proof of Proposition 2.6

To compare the price and profit between joint learning and uniform pricing, the parameters are in the joint set of the available range, which is $0<\lambda<\frac{4+2\theta}{3+2\theta}$ for any $\theta>0$.

First period price

$$p_{i1}^{J*}-p_{it}^{U*}=\frac{(\theta+1)\lambda\left(\theta\left(2\theta^2+11\theta-4\lambda+16\right)-2\lambda+4\right)(a-c-k)}{(4+2\theta(1-\lambda)-3\lambda)((\theta+2)^3-4(\theta+1)\lambda^2)}$$

The key term is $\theta\left(2\theta^2+11\theta-4\lambda+16\right)-2\lambda+4$, which is positive in the available range.

Second period price

$$p_{i2}^{J*}-p_{it}^{U*}=\frac{(\theta+1)\lambda\left(2(\theta+1)(\theta(2\theta+7)+2)\lambda-(\theta+2)^2(2\theta+1)\right)(a-c-k)}{(4+2\theta(1-\lambda)-3\lambda)\left((\theta+2)^3-4(\theta+1)\lambda^2\right)}$$

The key term is $2(\theta+1)(\theta(2\theta+7)+2)\lambda-(\theta+2)^2(2\theta+1)$, which is negative when $0<\lambda<\frac{2\theta^3+9\theta^2+12\theta+4}{4\theta^3+18\theta^2+18\theta+4}$. The threshold at the RHS is less than $\frac{4+2\theta}{3+2\theta}$.

Total profit

$$
\begin{aligned}
&\pi_i^{J*} - \pi_i^{U*} \\
&= 2(\theta+1)(a-c-k)^2 \\
&\times \Big(\frac{-4(\theta+1)\lambda^3 + 2(\theta+1)\left(\theta(\theta+2)^2-1\right)\lambda^2 + (\theta(2\theta+5)+4)(\theta+2)^2\lambda + (\theta+2)^4}{\left((\theta+2)^3 - 4(\theta+1)\lambda^2\right)^2} \\
&+ \frac{2(\lambda-2)}{(2\theta(\lambda-1)+3\lambda-4)^2}\Big) \\
&= 2\lambda(\theta+1)(a-c-k)^2 \\
&\times \Big(\frac{2\theta^2(\theta+2)^4 - 4(\theta+1)(2\theta+1)^2\lambda^4 + 2(\theta+1)(\theta(\theta(\theta(4\theta(\theta+7)+73)+96)+48)+7)\lambda^3}{\left((\theta+2)^3-4(\theta+1)\lambda^2\right)^2(2\theta(\lambda-1)+3\lambda-4)^2} \\
&+ \frac{-\theta(\theta(2\theta+5)(4\theta(\theta+3)+11)+10)(\theta+2)\lambda^2 - (\theta(\theta(4\theta(\theta+7)+51)+28)+4)(\theta+2)^2\lambda}{\left((\theta+2)^3-4(\theta+1)\lambda^2\right)^2(2\theta(\lambda-1)+3\lambda-4)^2}\Big)
\end{aligned}
$$

The key term is $M(\lambda) = 2\theta^2(\theta+2)^4 - 4(\theta+1)(2\theta+1)^2\lambda^4 + 2(\theta+1)(\theta(\theta(\theta(4\theta(\theta+7)+73)+96)+48)+7)\lambda^3 - \theta(\theta(2\theta+5)(4\theta(\theta+3)+11)+10)(\theta+2)\lambda^2 - (\theta(\theta(4\theta(\theta+7)+51)+28)+4)(\theta+2)^2\lambda$. We can check $M(0) = 2\theta^2(\theta+2)^4 > 0$ and $M(\frac{2\theta+4}{2\theta+3}) = -\frac{2(\theta+1)(\theta+2)^3(\theta(4\theta(\theta+5)+17)+2)^2}{(2\theta+3)^4} < 0$. Furthermore, we claim that $M(\lambda)$ firstly decreases and then increases with λ.

$$
\begin{aligned}
\frac{dM(\lambda)}{d\lambda} &= -16(\theta+1)(2\theta+1)^2\lambda^3 + 6(\theta+1)(\theta(\theta(\theta(4\theta(\theta+7)+73)+96)+48)+7)\lambda^2 \\
&\quad - 2\theta(\theta+2)(\theta(2\theta+5)(4\theta(\theta+3)+11)+10)\lambda - (\theta+2)^2(\theta(\theta(4\theta(\theta+7)+51)+28)+4)
\end{aligned}
$$

We find $\frac{dM(\lambda)}{d\lambda}|_{\lambda=0} < 0$ and $\frac{dM(\lambda)}{d\lambda}|_{\lambda=\frac{2\theta+4}{2\theta+3}} > 0$. Moreover, $\frac{d^2M(\lambda)}{d\lambda^2} = -48(\theta+1)(2\theta\lambda+\lambda)^2 + 12(\theta+1)(\theta(\theta(\theta(4\theta(\theta+7)+73)+96)+48)+7)\lambda - 2\theta(\theta+2)(\theta(2\theta+5)(4\theta(\theta+3)+11)+10)$ is negative when $0 < \lambda < \frac{4\theta^5+28\theta^4+73\theta^3+96\theta^2+48\theta+7}{8(2\theta+1)^2} - \frac{\sqrt{\frac{48\theta^{11}+720\theta^{10}+4776\theta^9+18416\theta^8+45659\theta^7+76123\theta^6+87200\theta^5+68354\theta^4+35426\theta^3+11360\theta^2+2003\theta+147}{(\theta+1)(2\theta+1)^4}}}{8\sqrt{3}}$ and positive otherwise. Hence, $\frac{dM(\lambda)}{d\lambda}$ is first negative and then positive. That is, $M(\lambda)$ first decreases and then increases with λ. Therefore, there exists a threshold $\lambda_2(\theta)$, $0 < \lambda_2(\theta) < \frac{2\theta+4}{2\theta+3}$, such that when $0 < \lambda \leq \lambda_2(\theta)$, $M(\lambda)$ is positive.

Proof of Proposition 2.7

We could derive the equilibrium results by backward induction. Given the first order production quantities, the profit function of the second period is given by:

$$\pi_{i2}^m = (p_{i2} - (c - \lambda(q_{i1} + q_{j1}) + k))q_{i2}$$

The second-period equilibrium price is the same as the base model $p_{i2}^{m*}(p_{i1}, p_{j1}) = \frac{a+(1+\theta)(c+k-\lambda(2a-p_{i1}+p_{j1}))}{2+\theta}$. For myopic OEMs, the first period objective is the sole first profit given as follows:

$$\pi_i^m(p_{i1}, p_{j1}) = (p_{i1} - (c + k))q_{i1}$$

The first-period equilibrium price is the same as the benchmark: $p_{i1} = \frac{a+(1+\theta)(c+k)}{2+\theta}$ with the demand $q_{i1} = \frac{(1+\theta)(a-c-k)}{2+\theta}$. Substitute the equilibrium prices into the profit function, the total profit of the OEMs is as follows:

$$\pi_i^{m*} = \frac{2(a-c-k)^2(1+\theta)((2+\theta)^2 + 2\lambda(1+\theta)(2+\theta) + 2(1+\theta)^2\lambda^2}{(2+\theta)^4}$$

Take the first order derivative with respect to θ, we can get

$$\frac{d\pi_i^{m*}}{d\theta} = -\frac{2\left(2(\theta-2)(\theta+1)^2\lambda^2 + 2(\theta-1)(\theta+1)(\theta+2)\lambda + \theta(\theta+2)^2\right)(a-c-k)^2}{(\theta+2)^5}$$

The key term is a quadratic function of θ, $M(\theta) = -(1+2\lambda+2\lambda^2)\theta^3 - 4(1+\lambda)\theta^2 + 2(-2+\lambda+3\lambda^2)\theta + 4\lambda + 4\lambda^2$. When $\theta = 0$, $M(\theta) > 0$ for any $\lambda > 0$. With the increasing of θ, $M(\theta)|_{\theta\to\infty} < 0$.

$$\frac{dM(\theta)}{d\theta} = -3\theta^2(2\lambda(\lambda+1)+1) - 8\theta(\lambda+1) + 6\lambda^2 + 2\lambda - 4$$

We find that $\frac{dM(\theta)}{d\theta}|_{\theta=0} = -4 + 2\lambda + 6\lambda^2$ and $\frac{dM(\theta)}{d\theta}|_{\theta\to\infty} < 0$. That is, $M(\theta)$ either firstly increase and then decreases with θ or decreases with θ. Therefore, there exists a threshold $\theta_2(\lambda)$ such that $M(\theta)$ is positive when $0 < \theta < \theta_2(\lambda)$ and negative when $\theta > \theta_2(\lambda)$.

Proof of Proposition 2.8

To compare the myopic pricing and differential pricing, we assume that $0 < \lambda < \frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}$ for any $\theta > 0$. First period price

The first period price is the same as the case without learning. Therefore the first period with myopic pricing is higher than the first period price with joint learning.

Second period price

$$p_{i2}^{J*} - p_{i2}^{M*} = \frac{4(\theta+1)^2\lambda^2(2(\theta+1)\lambda+\theta+2)(a-c-k)}{4(\theta+1)(\theta+2)^2\lambda^2 - (\theta+2)^5}$$

The key term $4(\theta+1)(\theta+2)^2\lambda^2 - (\theta+2)^5$ is negative in the whole available range.

Total profit

$$\pi_i^{J*} - \pi_i^{M*} = 2(\theta+1)(a-c-k)^2$$
$$\times \Big(\frac{-4(\theta+1)\lambda^3 + 2(\theta+1)\left(\theta(\theta+2)^2-1\right)\lambda^2 + (\theta(2\theta+5)+4)(\theta+2)^2\lambda + (\theta+2)^4}{\left((\theta+2)^3-4(\theta+1)\lambda^2\right)^2}$$
$$-\frac{2(\theta+1)^2\lambda^2 + 2(\theta+1)(\theta+2)\lambda + (\theta+2)^2}{(\theta+2)^4}\Big)$$
$$= \frac{2\lambda(\theta+1)(a-c-k)^2}{\left((\theta+2)^3-4(\theta+1)\lambda^2\right)^2(\theta+2)^4} \times \left(-2(\theta+1)\left(\theta^2-3\right)(\theta+2)^4\lambda - 32(\theta+1)^4\lambda^5\right.$$
$$-32(\theta+1)^3(\theta+2)\lambda^4 + 16(\theta+1)^2(\theta(\theta+3)+1)(\theta+2)^2\lambda^3 + 4(\theta+1)(4\theta+3)(\theta+2)^4\lambda^2$$
$$\left.-\theta(\theta+2)^6\right)$$

The key term is given by $M(\lambda) = -2(\theta+1)\left(\theta^2-3\right)(\theta+2)^4\lambda - 32(\theta+1)^4\lambda^5 - 32(\theta+1)^3(\theta+2)\lambda^4 + 16(\theta+1)^2(\theta(\theta+3)+1)(\theta+2)^2\lambda^3 + 4(\theta+1)(4\theta+3)(\theta+2)^4\lambda^2 - \theta(\theta+2)^6$. We find that $M(0) = -\theta(2+\theta)^6 > 0$ and $M(\frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}) = (\theta+1)^2(\theta+2)^4\left(3\sqrt{\frac{(\theta+2)^3}{\theta+1}} + \theta\left(3\sqrt{\frac{(\theta+2)^3}{\theta+1}} + \theta\left(\sqrt{\frac{(\theta+2)^3}{\theta+1}}+2\right)+8\right)+8\right) > 0$. Take the first order derivative with λ we get:

$$\frac{dM(\lambda)}{d\lambda} = -(\theta+2)^4\left(\theta^2-3\right) - 80(\theta+1)^3\lambda^4 - 64(\theta+1)^2(\theta+2)\lambda^3$$
$$+ 24(\theta+1)(\theta(\theta+3)+1)(\theta+2)^2\lambda^2 + 4(4\theta+3)(\theta+2)^4\lambda$$

We find that $\frac{dM(\lambda)}{d\lambda}|_{\lambda=0} = -(\theta+2)^4\left(\theta^2-3\right)$ and $\frac{dM(\lambda)}{d\lambda}|_{\lambda=\frac{2+\theta}{2}\sqrt{\frac{2+\theta}{1+\theta}}} = (\theta+2)^4\left(\theta(\theta(\theta+4)+2) - 2\sqrt{\frac{(\theta+2)^3}{\theta+1}} - 5\right)$. Furthermore, when $0 < \theta \le 1.38$, $\frac{dM(\lambda)}{d\lambda}$ is firstly positive and then negative that is, $M(\lambda)$ firstly increases and then decreases λ. When $1.38 < \theta \le 1.73$, $\frac{dM(\lambda)}{d\lambda}$ is always positive that is, $M(\lambda)$ increases with λ. When $\theta > 1.73$, $\frac{dM(\lambda)}{d\lambda}$ is firstly negative and then positive that is $M(\lambda)$ firstly increases and then decreases with λ. In each case, there exists a threshold $\lambda_3(\theta)$ such that when $0 < \lambda \le \lambda_3(\theta)$, $M(\lambda)$ is negative and when $\lambda > \lambda_3(\theta)$, $M(\lambda)$ is positive.

Equilibrium results when the CM has pricing power

Since OEMs function as the Stackelberg leader, the CM determines the margin observing the OEMs' decisions. We can derive the equilibrium by backward induction. In the second period, the CM determines the markup given the OEM's second period margin.

$$\Pi_2 = q_{12}k_2 + q_{22}k_2$$

The profit function is concave in k_2. The best response function of k_2 is given by $k_2 = \frac{1}{4}(2a - 2c - m_{12} - m_{22} + 2\lambda(q_{11} + q_{21}))$. Anticipating the response of CM, OEMs determine the retail margin of the second period simultaneously.

$$\pi_{i2} = \frac{1}{4}m_{12}(2a - 2c - 4\theta m_{12} - 3m_{12} + 4\theta m_{22} + m_{22} + 2\lambda(q_{11} + q_{21}))$$

The OEM's profit function is concave in the retail margin. The intersection of the response functions of the OEMs' gives the second period equilibrium margin

$$m_{12}^* = \frac{2(a - c + \lambda(q_{11} + q_{21}))}{4\theta + 5}$$

Hence the CM's margin in the second period is $k_2 = \frac{(4\theta+3)(a-c+\lambda(q_{11}+q_{21}))}{8\theta+10}$. The second period price could be calculated as $p_{i2} = \frac{a(4\theta+7)+(4\theta+3)(c-\lambda(q_{11}+q_{21}))}{8\theta+10}$ and the corresponding demand is given by $q_{i2} = \frac{(4\theta+3)(a-c+\lambda(q_{11}+q_{21}))}{8\theta+10}$. OEM's second period profit is given by

$$\pi_{i2} = \frac{(4\theta + 3)(a - c + \lambda(q_{11} + q_{21}))^2}{(4\theta + 5)^2}$$

CM's second period profit is given by

$$\Pi_2^* = \frac{2(4\theta + 3)^2(a - c + \lambda(q_{11} + q_{21}))^2}{(8\theta + 10)^2}$$

The decision sequence in the first period is similar. Observing the OEMs' retail margin, CM determines the margin to maximize the two-period total profit.

$$\Pi = (q_{11} + q_{21}) + \Pi_2^*.$$

When $0 < \lambda < \frac{5+4\theta}{3+4\theta}$ for any $\theta > 0$, the CM's total profit is concave in the first period wholesale margin. The best response function of CM is given by

$$k_1 = \frac{2(4\theta+3)^2\lambda(a-c) + 2(4\theta+3)^2\lambda^2(2a-2c-m_{11}-m_{21}) - (4\theta+5)^2(2a-2c-m_{11}-m_{21})}{4(4\theta+3)^2\lambda^2 - 4(4\theta+5)^2}$$

In anticipating the CM's response, OEMs determine the first period retail margin to simultaneously to maximize the two-period total profit

$$\pi_i = q_{11}m_{11} + \pi_{i2}^*$$

The intersection of the best response functions of the OEMs gives the equilibrium first period retail margin

$$m_{i1}^* = \frac{2(a-c)\left(-\lambda^3(4\theta+3)^4 - (4\theta+5)^3(4\theta+3)\lambda^2 + (4\theta+1)(4\theta+5)^2(4\theta+3)\lambda + (4\theta+5)^4\right)}{2(2\theta+1)(4\theta+3)^4\lambda^4 - (4\theta+3)(4\theta(8\theta+13)+25)(4\theta+5)^2\lambda^2 + (4\theta+5)^5}$$

Substituting into the profit function we can obtain the equilibrium total profit.

Appendix B

Proofs in Chapter 3

Proof of Proposition 3.2

We solve the problem by backward induction. Given the wholesale price and trade credit interest rate, the retailer determines the order quantity to maximize his expected profit. In the bankruptcy risk range $q_2 \geq \frac{A-w_1(1+r)-2\bar{z}}{\gamma}$, the first order condition leads to two roots $\frac{1}{3}(A-w(1+r)-\gamma q_{3-i}+\bar{z})$ and $A-w(1+r)-\gamma q_{3-i}+\bar{z}$. Checking the second order condition, $\frac{\partial^2 \pi_i^{NN}}{\partial q_i^2}|_{q_i=\frac{1}{3}(A-w(1+r)-\gamma q_{3-i}+\bar{z})} < 0$ and $\frac{\partial^2 \pi_i^{NN}}{\partial q_i^2}|_{q_i=A-w(1+r)-\gamma q_{3-i}+\bar{z}} > 0$, only the first root is kept. In the no bankruptcy risk range $q_2 < \frac{A-w_1(1+r)-2\bar{z}}{\gamma}$, the response function of the retailer is given by $\frac{1}{2}(A-w_1(1+r)-\gamma q_{3-i})$. The interaction of either retailer's response function gives the second-stage equilibrium $q_i = \frac{A-w(1+r)+\bar{z}}{3+\gamma}$ when $\bar{z} \geq \frac{A-w(1+r)}{2+\gamma}$ and $q_i = \frac{A-w(1+r)}{2+\gamma}$ when $\bar{z} < \frac{A-w(1+r)}{2+\gamma}$. The resulting critical threshold is $\hat{z}(w,r) = \frac{(1+\gamma)\bar{z}-2(A-w(1+r))}{3+\gamma}$.

Anticipating the retailer's response, the supplier determines the wholesale price and trade credit interest rate to maximize the expected profit. The term $w(1+r)$ is inseparable could be treated as adjusted wholesale price. We can obtain $w(1+r) = A + 2\bar{z} - \sqrt{(3A+\bar{z})\bar{z}}$ in the no bankruptcy risk range and $w(1+r) = \frac{1}{3}((\gamma(4+\gamma)+6)\bar{z} - (3+\gamma)\sqrt{\bar{z}\,(3A+(1+\gamma)^2\bar{z})}) + A$ in the bankruptcy risk range. The profit of the supplier is given by $\frac{(A-c)^2}{2(2+\gamma)}$ in the no bankruptcy risk range and $\frac{(\Delta_1-(1+\gamma)\bar{z})^3}{27\bar{z}}$ in the bankruptcy risk range, here $\Delta_1 = \sqrt{3A\bar{z}+(1+\gamma)^2\bar{z}^2}$. We can further check that $\frac{(\Delta_1-(1+\gamma)\bar{z})^3}{27\bar{z}} \geq \frac{(A-c)^2}{2(2+\gamma)}$ when $\bar{z} \geq \frac{A}{4(1+\gamma)^2}\left((5+\gamma)\sqrt{\frac{5+\gamma}{2+\gamma}}-7+\gamma\right)$, which gives the critical threshold for the bankruptcy risk range. The equilibrium selling quantities and retailers' profits could be derived by substituting the adjusted wholesale price.

Proof of Corollary 3.3

By comparing the selling quantities in the bankruptcy risk range and no bankruptcy risk range, we can obtain:

$$q_i^{NN} - q_i^{YY} = -\frac{A}{4+2\gamma} + \sqrt{3A\bar{z}+(1+\gamma)^2\bar{z}^2} - \frac{1}{3}(1+\gamma)\bar{z}$$

The RHS is increasing with $\bar{z}$ and we can check that the RHS equals zero when $\bar{z} = \frac{3A}{4(2+\gamma)}$.

Proof of Corollary 3.4

Take the first order derivative with $\bar{z}$ for the retailer's profit in the bankruptcy risk range, we can get:

$$\frac{d\pi_i^{NN}}{d\bar{z}} = \frac{\left(\sqrt{3A+(\gamma+1)^2\bar{z}} - 3(\gamma+1)\sqrt{\bar{z}}\right)\left(\sqrt{3A+(\gamma+1)^2\bar{z}} - (\gamma+1)\sqrt{\bar{z}}\right)^3}{54\sqrt{3A\bar{z}+(\gamma+1)^2\bar{z}^2}}$$

When $0 < \gamma < 0.4$, π_i^{NN} increases with $\bar{z}$ when $\hat{\gamma}A < \bar{z} < \frac{3A}{8\gamma^2+16\gamma+8}$ and then decreases with $\bar{z}$ when $\bar{z} \geq \frac{3A}{8\gamma^2+16\gamma+8}$. We can further check $\pi_i^{NN}(\bar{z} = \frac{3A}{8\gamma^2+16\gamma+8}) < \pi_i^{YY}$. When $\gamma \geq 0.4$, π_i^{NN} decreases with $\bar{z}$ in the bankruptcy risk range. We can further check $\pi_i^{NN}(\bar{z} = \hat{\gamma}A) < \pi_i^{YY}$. This completes the proof.

Proof of Proposition 3.5

We solve the problem by backward induction. In the second stage, retailers determine the selling quantities given the wholesale price and trade credit interest rate. Retailer 1 falls into the bankruptcy risk range when $q_2 \geq \frac{A-w_1(r+1)-2\bar{z}}{\gamma}$. The first order condition gives two roots for q_1: $\frac{1}{3}(A - \gamma q_2 - w_1(1+r) + \bar{z})$ and $A - \gamma q_2 - w_1(1+r) + \bar{z}$. By checking the second order condition, $\frac{\partial^2 \pi_1^{NY}}{\partial q_1^2}|_{q_1=\frac{1}{3}(A-w_1(1+r)-\gamma q_2+\bar{z})} < 0$ and $\frac{\partial^2 \pi_1^{NY}}{\partial q_1^2}|_{q_1=A-w_1(1+r)-\gamma q_2+\bar{z}} > 0$, the best response function of retailer 1 is given by $q_1 = \frac{1}{3}(A - \gamma q_2 - w_1(1+r) + \bar{z})$. Retailer 1 is in the no bankruptcy risk range when $q_2 < \frac{A-w_1(r+1)-2\bar{z}}{\gamma}$, the best response function is given by $q_1 = \frac{1}{2}(A - \gamma q_2 - w_1(1+r))$, which is independent with $\bar{z}$.

For retailer 2, the best response function is derived from the first order condition as $q_2 = \frac{1}{2}(A - \gamma q_1 - w_2)$ since the profit function is concave in q_2. The second-stage equilibrium selling quantities are derived by the intersection of the best response functions.

$$q_1^{NY}(w_1, w_2, r) = \begin{cases} \frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2+2\bar{z}}{6-\gamma^2} & \text{if } \bar{z} \geq \frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2} \\ \frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2} & \text{if } \bar{z} < \frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2} \end{cases}$$

$$q_2^{NY}(w_1, w_2, r) = \begin{cases} \frac{(3-\gamma)A+\gamma w_1(1+r)-3w_2-\gamma\bar{z}}{6-\gamma^2} & \text{if } \bar{z} \geq \frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2} \\ \frac{(2-\gamma)A+\gamma w_1(1+r)-2w_2}{4-\gamma^2} & \text{if } \bar{z} < \frac{(2-\gamma)A-2w_1(1+r)+\gamma w_2}{4-\gamma^2} \end{cases}$$

Anticipating the selling quantities, the supplier determines the wholesale price and trade credit interest rate to maximize the expected profit. The term $w_1(1+r)$ is inseparable could be treated as adjusted wholesale price for retailer 1. The first order condition gives the optimal adjusted wholesale price $\frac{1}{12}(3(4-\gamma)A + (24 - 8\gamma^2 + \gamma^4)\bar{z} - (6-\gamma^2)\Delta_2)$ in the bankruptcy risk range and $\frac{A}{2}$ in the no bankruptcy risk range.

The optimal wholesale price for retailer 1 is given by $\frac{A}{2}$. Substituting the wholesale price and trade credit interest rate into the supplier's profit function, we can obtain supplier's profit as $\frac{1}{216}(27A^2 + 6A(2-\gamma)(2\Delta_2 - 3(2-\gamma^2)\bar{z}) + 2(2-\gamma^2)^2(\Delta_2 - (2-\gamma^2)\bar{z})\bar{z})$ in the bankruptcy risk range and $\frac{A^2}{2(\gamma+2)}$ in the no bankruptcy risk range, here $\Delta_2 = \sqrt{\bar{z}(6A(2-\gamma) + \bar{z}(2-\gamma^2)^2)}$. We can further check that $\frac{1}{216}(27A^2 + 6A(2-\gamma)(2\Delta_2 - 3(2-\gamma^2)\bar{z}) + 2(2-\gamma^2)^2(\Delta_2 - (2-\gamma^2)\bar{z})\bar{z}) \geq \frac{A^2}{2(\gamma+2)}$ when $\bar{z} \geq \frac{A}{4(2-\gamma^2)^2}((10-\gamma^2)\sqrt{\frac{(2-\gamma)(10-\gamma^2)}{2+\gamma}} - (2-\gamma)(14+\gamma^2))$, which is the critical threshold of bankruptcy risk range. The equilibrium selling quantities and retailer's profit can be derived by substituting into the wholesale price and trade credit interest rate.

Proof of Corollary 3.6

By comparing the selling quantity of retailer 1 in the bankruptcy risk range and no bankruptcy risk range, we obtain:

$$q_1^{NY} - q_i^{YY} = \frac{(2+\gamma)(\sqrt{6A\bar{z}(2-\gamma) + (2-\gamma^2)^2\bar{z}^2} - (2-\gamma^2)\bar{z}) - 3A}{6(2+\gamma)}$$

We can check that the RHS is increasing with $\bar{z}$:

$$\frac{d(q_1^{NY} - q_i^{YY})}{d\bar{z}} = \frac{1}{6}\left(\frac{(2-\gamma^2)^2\,\bar{z} + 3A(2-\gamma)}{\sqrt{6A\bar{z}(2-\gamma) + (2-\gamma^2)^2\bar{z}^2}} + \gamma^2 - 2\right) > 0$$

and equals zero when $\bar{z} = \frac{3A}{4(2+\gamma)}$. By comparing the selling quantity of retailer 2 in the bankruptcy risk range and no bankruptcy risk range, we get:

$$q_2^{NY} - q_i^{YY} = -\frac{\gamma((2+\gamma)(\sqrt{6A\bar{z}(2-\gamma) + (2-\gamma^2)^2\bar{z}^2} - (2-\gamma^2)\bar{z}) - 3A)}{12(2+\gamma)}$$

We can check that the RHS is decreasing with $\bar{z}$:

$$\frac{d(q_2^{NY} - q_i^{YY})}{d\bar{z}} = -\frac{1}{12}\gamma\left(\frac{\left(2-\gamma^2\right)^2\bar{z} + 3A(2-\gamma)}{\sqrt{6A\bar{z}(2-\gamma) + (2-\gamma^2)^2\bar{z}^2}} + \gamma^2 - 2\right) < 0$$

and $q_2^{NY} - q_i^{YY}$ equals zero when $\bar{z} = \frac{3A}{4(2+\gamma)}$. This completes the proof.

Proof of Corollary 3.7

For retailer 1, we compare his profit in bankruptcy risk range and no bankruptcy risk range as follows:

$$\pi_1^{NY} - \pi_i^{YY} = \frac{\left(\sqrt{6A\bar{z}(2-\gamma) + (\gamma-2)^2\bar{z}^2} - \left(2-\gamma^2\right)\bar{z}\right)^3}{216\bar{z}} - \frac{A^2}{4(\gamma+2)^2}$$

We can check that the RHS is increasing with $\bar{z}$ when $\tilde{\gamma} \leq \bar{z} < \frac{6A-3A\gamma}{4\gamma^4-16\gamma^2+16}$ and decreases with $\frac{6A-3A\gamma}{4\gamma^4-16\gamma^2+16} \leq \bar{z} < A$. We can further check when $\bar{z} = \frac{6A-3A\gamma}{4\gamma^4-16\gamma^2+16}$, the RHS is negative.

For retailer 2, we compare his profit in bankruptcy risk range and no bankruptcy risk range as follows:

$$\pi_2^{NY} - \pi_i^{YY} = \frac{1}{144}\left(3A + \gamma(2-\gamma^2)\bar{z} - \gamma\sqrt{6A\bar{z}(2-\gamma) + (\gamma-2)^2\bar{z}^2}\right)^2 - \frac{A^2}{4(\gamma+2)^2}$$

We can check that the RHS is decreasing with $\bar{z}$ and RHS equals zero when $\bar{z} = \frac{3A}{4\gamma+8}$.

Proof of Corollary 3.8

To compare the critical ratio in (N, N) and (N, Y) equilibrium, we obtain:

$$\hat{\gamma} - \tilde{\gamma} = \frac{\gamma + (\gamma+5)\sqrt{\frac{\gamma+5}{\gamma+2}} - 7}{4(\gamma+1)^2} - \frac{(\gamma-2)\left(\gamma^2+14\right) + \left(10-\gamma^2\right)\sqrt{\frac{(\gamma-2)(\gamma^2-10)}{\gamma+2}}}{4\left(\gamma^2-2\right)^2}$$

We can check that the RHS is increasing with γ for $0 < \gamma < 1$. When $\gamma = 0$, $\hat{\gamma} = \tilde{\gamma}$. Therefore, $\hat{\gamma} > \tilde{\gamma}$ for any γ when $0 < \gamma < 1$. This completes the proof.

Proof of Corollary 3.9

To compare the adjusted wholesale prices in the bankruptcy risk range of (N, N) and (N, Y), we have:

$$\begin{aligned} & w_1^{NY}(1 + r_1^{NY}) - w_i^{NN}(1 + r_i^{NN}) \\ & = \frac{1}{12}\left(3(4-\gamma)A + (24 - 8\gamma^2 + \gamma^4)\bar{z} - (6-\gamma^2)\Delta_2\right) - \frac{1}{3}\left(3A + (\gamma^2 + 4\gamma + 6)\bar{z} - (3+\gamma)\Delta_1\right)\right) \end{aligned}$$

We can check that the RHS is increasing with $\bar{z}$ when $\hat{\gamma}A \leq \bar{z} < \frac{3A}{4\gamma+8}$ and decreasing with $\bar{z}$ when $\frac{3A}{4\gamma+8} < \bar{z} < A$. We can further check when $\bar{z} = \frac{3A}{4\gamma+8}$, $w_1^{NY}(1 + r_1^{NY}) - w_i^{NN}(1 + r_i^{NN}) = \frac{A\left(\gamma^2-6\right)^2}{8(\gamma+2)}$ is also negative. Therefore, we have $w_1^{NY}(1 + r_1^{NY}) < w_i^{NN}(1 + r_i^{NN})$ when $\hat{\gamma}A \leq \bar{z} < A$.

Proof of Corollary 3.10

By comparing retailer 1's selling quantities in the bankruptcy risk range of (N, N) and (N, Y), we have:

$$q_1^{NY} - q_i^{NN} = \frac{1}{6}\left(\Delta_2 - (2-\gamma^2)\bar{z}\right) - \frac{\Delta_1 - (1+\gamma)\bar{z}}{3}$$

We can check that the RHS is increasing with $\bar{z}$ in the bankruptcy risk range and equals zero when $\bar{z} = \frac{3A}{4(\gamma+2)}$. Therefore, when $\hat{\gamma} < \gamma < \frac{3A}{4(\gamma+2)}$, $q_1^{NY} < q_i^{NN}$ and when $\frac{3A}{4(\gamma+2)} \leq \bar{z} < Z$, $q_1^{NY} \geq q_i^{NN}$. By comparing retailer 2's selling quantities in the bankruptcy risk range of (N, N) and (N, Y), we have:

$$q_2^{NY} - q_i^{NN} = \frac{1}{12}\left(3A + (2\gamma - \gamma^3)\bar{z} - \gamma\Delta_2\right) - \frac{\Delta_1 - (1+\gamma)\bar{z}}{3}$$

We can check that the RHS is decreasing with $\bar{z}$ in the bankruptcy risk range and equals zero when $\bar{z} = \frac{3A}{4(\gamma+2)}$. Hence, when $\hat{\gamma} < \gamma < \frac{3A}{4(\gamma+2)}$, $q_2^{NY} > q_i^{NN}$ and when $\frac{3A}{4(\gamma+2)} \leq \bar{z} < Z$, $q_1^{NY} \leq q_i^{NN}$. This completes the proof.

Proof of Corollary 3.11

To compare the supplier's profit in the bankruptcy risk range of (N, N) and (N, Y), we have

$$\begin{aligned}
&\pi_s^{NY} - \pi_s^{NN} \\
&= \frac{1}{216}\left(27A^2 + 6A(2-\gamma)(2\Delta_2 - 3(2-\gamma^2)\bar{z}) + 2(2-\gamma^2)^2(\Delta_2 - (2-\gamma^2)\bar{z})\bar{z}\right) \\
&- \frac{2}{27}(\Delta_1 - (1+\gamma)\bar{z})\left(6A + (1+\gamma)^2\bar{z} - (1+\gamma)\Delta_1\right)
\end{aligned}$$

Both π_s^{NY} and π_s^{NN} are increasing with $\bar{z}$. Take the first order derivative with the RHS, we can obtain:

$$\begin{aligned}
&\frac{d(\pi_s^{NY} - \pi_s^{NN})}{d\bar{z}} = \frac{1}{216}\left(4\left(\gamma^2-2\right)^2\left(\Delta_2 + \left(\gamma^2-2\right)\bar{z}\right) - \frac{6A(\gamma-2)\left(\Delta_2 + 3\left(\gamma^2-2\right)\bar{z}\right)}{\bar{z}}\right) \\
&- \frac{2}{27}\left(\frac{3A + 2(\gamma+1)^2\bar{z}}{2\Delta_1} - \gamma - 1\right)\left((\gamma+1)\left(-\Delta_1 + \gamma\bar{z} + \bar{z}\right) + 6A\right) \\
&- \frac{2}{27}(\gamma+1)\left(\Delta_1 - (\gamma+1)\bar{z}\right)\left(-\frac{3A + 2(\gamma+1)^2\bar{z}}{2\Delta_1} + \gamma + 1\right)
\end{aligned}$$

When $0 < \gamma < 0.974$, the RHS is decreasing with $\bar{z}$. When $0.974 \leq \gamma < 1$, the RHS firstly decreases and then increases with $\bar{z}$. We can check when $\bar{z} = \hat{\gamma}A$, RHS is positive while when $\bar{z} = A$, RHS is negative. Therefore, in either case, there exists a critical threshold $\bar{z}_0$ such than when $\bar{z} < \bar{z}_0$ the RHS is positive and when $\bar{z} \geq \bar{z}_0$ the RHS is negative. This completes the proof.

Proof of Corollary 3.12

Since the cases for both retailers are symmetrical, without lose of generality, we first consider the effect when the focal retailer (retailer 2)'s financial status changes from N to Y, given the competitor (retailer 1)'s financial status is N. That is, we compare π_2^{NN} and π_2^{NY}.

When $0 < \bar{z} < \tilde{\gamma}A$, $\pi_2^{NN} = \pi_2^{NY} = \pi_i^{YY}$. In both cases (N, N) and (N, Y), retailer 2 is in the no bankruptcy risk range and retailer 2's profit equals to the benchmark.

When $\tilde{\gamma}A \leq \bar{z} < \hat{\gamma}A$, $\pi_2^{NY} > \pi_2^{NN} = \pi_i^{YY}$. Retailer 2 falls into the bankruptcy risk range in (N, Y) but still in the no bankruptcy risk range of (N, N).

When $\hat{\gamma}A \leq \bar{z} < A$, in both cases, retailer 2 falls into t the bankruptcy risk range. We can also check that $\pi_2^{NY} > \pi_2^{NN}$.

Now given the competitor (retailer 2)'s financial status is Y, we consider the effect when the focal retailer (retailer 1)'s financial status changes from N to Y. We compare π_1^{NY} and π_1^{YY}.

When $0 < \bar{z} < \hat{\gamma}A$, $\pi_1^{NY} = \pi_1^{YY}$ since the retailer 1 is in the no bankruptcy risk range of (N, Y).

When $\hat{\gamma}A \leq \bar{z} < A$, we can find that $\pi_1^{NY} < \pi_1^{YY}$ from Corollary 3.7.

Proof of Corollary 3.13

Given the focal retailer (retailer 1)'s financial status is N, we consider the effect when the competitor (retailer 2)'s financial status changes from N to Y. That is, we compare π_1^{NN} and π_1^{NY}.

When $0 < \bar{z} < \tilde{\gamma}A$, $\pi_1^{NN} = \pi_1^{NY}$ since retailer 1 is in the no bankruptcy risk range of both (N, N) and (N, Y).

When $\tilde{\gamma}A \leq \bar{z} < \hat{\gamma}A$, we can find that $\pi_1^{NN} > \pi_1^{NY}$ since retailer 1 is in the bankruptcy risk range of (N, Y) and the no bankruptcy risk range o f (N, N).

When $\hat{\gamma}A \leq \bar{z} < \frac{3A}{4(2+\gamma)}$, we can find that $\pi_1^{NN} > \pi_1^{NY}$. When $\frac{3A}{4(2+\gamma)} \leq \bar{z} < A$, we can find that $\pi_1^{NN} \leq \pi_1^{NY}$.

Given the focal retailer (retailer 2)'s financial status is Y, we consider the effect when the competitor (retailer 1)'s financial status changes from N to Y. That is, we compare π_2^{NN} and π_2^{YY}.

When $0 < \bar{z} < \tilde{\gamma}A$, $\pi_2^{NY} = \pi_2^{YY}$ since retailer 2 is in the no bankruptcy risk range of (N, Y).

When $\tilde{\gamma} \leq \bar{z} < \frac{3A}{4(2+\gamma)}$, we can find that $\pi_2^{NY} > \pi_2^{YY}$ from Corollary 3.7.

When $\frac{3A}{4(2+\gamma)} \leq \bar{z} < A$, we can find that $\pi_2^{NY} \leq \pi_2^{YY}$ from Corollary 3.7.

Appendix C

Proofs in Chapter 4

Stackelberg-manufacturer as the leader: Second-stage Analysis

- (Direct, Direct) Equilibrium

When manufacturer is the second-stage pricing game leader and both supply chains adopt direct collection, the equilibrium results are characterized as follows:

(1) The profit margins of retailer and manufacturer are given by

$$\bar{m}_i^{DD} = \frac{2B(\mu - (1-\beta)c)}{(8-6\beta)B - (1-\beta)\Delta^2} \text{ and } \bar{M}_i^{DD} = \frac{4B(\mu - (1-\beta)c)}{(8-6\beta)B - (1-\beta)\Delta^2};$$

(2) The collection rate is

$$\bar{\tau}_i^{DD} = \frac{\Delta(\mu - (1-\beta)c)}{(8-6\beta)B - (1-\beta)\Delta^2};$$

(3) The profits of retailer and manufacturer are given by

$$\bar{\pi}_i^{DD} = \frac{4B^2(\mu - (1-\beta)c)^2}{((8-6\beta)B - (1-\beta)\Delta^2)^2} \text{ and } \bar{\Pi}_i^{DD} = \frac{B(8B - \Delta^2)(\mu - (1-\beta)c)^2}{((8-6\beta)B - (1-\beta)\Delta^2)^2}.$$

- (Direct, Indirect) Equilibrium

When manufacturer is the second-stage pricing game leader, supply chain 1 adopts direct collection and supply chain 2 adopts indirect collection, the equilibrium results are characterized as follows:

(1) The profit margins of the retailers are given by

$$\bar{m}_1^{DI} = \frac{2B((4+3\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4} \text{ and}$$

$$\bar{m}_2^{DI} = \frac{B((8+6\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4}$$

(2) The profit margins of manufacturers are given by

$$\bar{M}_1^{DI} = \frac{4B((4+3\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4} \text{ and}$$

$$\bar{M}_2^{DI} = \frac{(4B-\Delta^2)((8+6\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2(2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4)}$$

(3) The collection rates are given by

$$\bar{\tau}_1^{DI} = \frac{\Delta((4+3\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4} \text{ and}$$

$$\bar{\tau}_2^{DI} = \frac{\Delta((8+6\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2(2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4)}$$

(4) The profits of retailers are given by

$$\bar{\pi}_1^{DI} = \left(\frac{2B((4+3\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4}\right)^2 \text{ and}$$

$$\bar{\pi}_2^{DI} = \frac{B(4B-\Delta^2)((8+6\beta)B-(1+\beta)\Delta^2)^2(\mu-(1-\beta)c)^2}{4(2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4)^2}$$

(5) The profits of manufacturers are given by

$$\bar{\Pi}_1^{DI} = \frac{B(8B-\Delta^2)((4+3\beta)B-(1+\beta)\Delta^2)^2(\mu-(1-\beta)c)^2}{(2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4)^2} \text{ and}$$

$$\bar{\Pi}_2^{DI} = \frac{B(4B-\Delta^2)((8+6\beta)B-(1+\beta)\Delta^2)^2(\mu-(1-\beta)c)^2}{2(2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4)^2}$$

- (Indirect, Indirect) Equilibrium

When manufacturer is the second-stage pricing game leader and both supply chains adopt indirect collection, the equilibrium results are characterized as follows:

(1) The profit margins of retailer and manufacturer are given by

$$\bar{m}_i^{II} = \frac{B(\mu-(1-\beta)c)}{(4-3\beta)B-(1-\beta)\Delta^2} \text{ and } \bar{M}_i^{II} = \frac{(4B-\Delta^2)(\mu-(1-\beta)c)}{2((4-3\beta)B-(1-\beta)\Delta^2)}$$

(2) The collection rate is given by

$$\bar{\tau}_i^{II} = \frac{\Delta(\mu-(1-\beta)c)}{2((4-3\beta)B-(1-\beta)\Delta^2)}$$

(3) The profits of retailer and manufacturer are given by

$$\bar{\pi}_i^{II} = \frac{B(4B-\Delta^2)(\mu-(1-\beta)c)^2}{4((4-3\beta)B-(1-\beta)\Delta^2)^2} \text{ and } \bar{\Pi}_i^{II} = \frac{B(4B-\Delta^2)(\mu-(1-\beta)c)^2}{2((4-3\beta)B-(1-\beta)\Delta^2)^2}$$

Stackelberg-retailer as the leader: Second-stage Analysis

- (Direct, Direct)

When retailer is the second-stage pricing game leader and both supply chains adopt direct collection, the equilibrium results are characterized as follows:

(1) The profit margins of retailer and manufacturer are given by

$$\tilde{m}_i^{DD} = \frac{(4B-\Delta^2)(\mu-(1-\beta)c)}{2((4-3\beta)B-(1-\beta)\Delta^2)} \text{ and } \tilde{M}_i^{DD} = \frac{B(\mu-(1-\beta)c)}{(4-3\beta)B-(1-\beta)\Delta^2};$$

(2) The collection rate is

$$\tilde{\tau}_i^{DD} = \frac{\Delta(\mu-(1-\beta)c)}{2((4-3\beta)B-(1-\beta)\Delta^2)};$$

(3) The profits of retailer and manufacturer are given by

$$\tilde{\pi}_i^{DD} = \frac{B(4B-\Delta^2)(\mu-(1-\beta)c)^2}{2((4-3\beta)B-(1-\beta)\Delta^2)^2} \text{ and } \tilde{\Pi}_i^{DD} = \frac{B(4B-\Delta^2)(\mu-(1-\beta)c)^2}{4((4-3\beta)B-(1-\beta)\Delta^2)^2}.$$

- (Direct, Indirect)

When manufacturer is the second-stage pricing game leader, supply chain 1 adopts direct collection and supply chain 2 adopts indirect collection, the equilibrium results are characterized as follows:

(1) The profit margins of the retailers are given by

$$\tilde{m}_1^{DI} = \frac{(4B-\Delta^2)((8+6\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2(2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4)} \text{ and}$$

$$\tilde{m}_2^{DI} = \frac{4B((4+3\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4}$$

(2) The profit margins of manufacturers are given by

$$\tilde{M}_1^{DI} = \frac{B((8+6\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4} \text{ and}$$

$$\tilde{M}_2^{DI} = \frac{2B((4+3\beta)B-(1+\beta)\Delta^2)(\mu-(1-\beta)c)}{2B^2(16-9\beta^2)-3B(4-3\beta^2)\Delta^2+(1-\beta^2)\Delta^4}$$

(3) The collection rates are given by

$$\tilde{\tau}_1^{DI} = \frac{\Delta((8+6\beta)B - (1+\beta)\Delta^2)(\mu - (1-\beta)c)}{2(2B^2(16-9\beta^2) - 3B(4-3\beta^2)\Delta^2 + (1-\beta^2)\Delta^4)} \text{ and}$$

$$\tilde{\tau}_2^{DI} = \frac{\Delta((4+3\beta)B - (1+\beta)\Delta^2)(\mu - (1-\beta)c)}{2B^2(16-9\beta^2) - 3B(4-3\beta^2)\Delta^2 + (1-\beta^2)\Delta^4}$$

(4) The profits of retailers are given by

$$\tilde{\pi}_1^{DI} = \frac{B(4B - \Delta^2)((8+6\beta)B - (1+\beta)\Delta^2)(\mu - (1-\beta)c)}{2(2B^2(16-9\beta^2) - 3B(4-3\beta^2)\Delta^2 + (1-\beta^2)\Delta^4)} \text{ and}$$

$$\tilde{\pi}_2^{DI} = \frac{B(8B - \Delta^2)((4+3\beta)B - (1+\beta)\Delta^2)^2(\mu - (1-\beta)c)^2}{(2B^2(16-9\beta^2) - 3B(4-3\beta^2)\Delta^2 + (1-\beta^2)\Delta^4)^2}$$

(5) The profits of manufacturers are given by

$$\tilde{\Pi}_1^{DI} = \frac{B(4B - \Delta^2)((8+6\beta)B - (1+\beta)\Delta^2)^2(\mu - (1-\beta)c)^2}{4(2B^2(16-9\beta^2) - 3B(4-3\beta^2)\Delta^2 + (1-\beta^2)\Delta^4)^2} \text{ and}$$

$$\tilde{\Pi}_2^{DI} = \frac{4B^2((4+3\beta)B - (1+\beta)\Delta^2)^2(\mu - (1-\beta)c)^2}{(2B^2(16-9\beta^2) - 3B(4-3\beta^2)\Delta^2 + (1-\beta^2)\Delta^4)^2}$$

- (Indirect, Indirect)

When retailer is the second-stage pricing game leader and both supply chains adopt indirect collection, the equilibrium results are characterized as follows:

(1) The profit margins of retailer and manufacturer are given by

$$\tilde{m}_i^{II} = \frac{4B(\mu - (1-\beta)c)}{(8-6\beta)B - (1-\beta)\Delta^2} \text{ and } \tilde{M}_i^{II} = \frac{2B(\mu - (1-\beta)c)}{(8-6\beta)B - (1-\beta)\Delta^2};$$

(2) The collection rate is

$$\tilde{\tau}_i^{II} = \frac{\Delta(\mu - (1-\beta)c)}{(8-6\beta)B - (1-\beta)\Delta^2};$$

(3) The profits of retailer and manufacturer are given by

$$\tilde{\pi}_i^{II} = \frac{B(8B-\Delta^2)(\mu-(1-\beta)c)^2}{((8-6\beta)B-(1-\beta)\Delta^2)^2} \text{ and } \tilde{\Pi}_i^{II} = \frac{4B^2(\mu-(1-\beta)c)^2}{((8-6\beta)B-(1-\beta)\Delta^2)^2}.$$

Proof of Proposition 4.3

The payoff matrix of the first stage is listed as follows:

	Direct	Indirect
Direct	$(\bar{\Pi}_1^{DD}, \bar{\Pi}_1^{DD})$	$(\bar{\Pi}_1^{DI}, \bar{\Pi}_2^{DI})$
Indirect	$(\bar{\Pi}_1^{ID}, \bar{\Pi}_2^{ID})$	$(\bar{\Pi}_1^{II}, \bar{\Pi}_2^{II})$

(1) Suppose (*Direct, Direct*) is the Nash equilibrium, then when manufacturer 1 chooses direct collection, it should also be optimal for manufacturer 2 to choose direct, that is $\Pi_2^{DD} > \Pi_2^{DI}$. Symmetrically, when manufacturer chooses direct collection, it should be optimal for manufacturer 1 to choose direct, that is, $\Pi_1^{DD} > \Pi_1^{ID}$. By symmetry, we have $\Pi_1^{DD} = \Pi_2^{DD}$ and $\Pi_1^{ID} = \Pi_2^{DI}$. Therefore, checking the condition for one of the inequality is sufficient.

$$\begin{aligned}
&\Pi_2^{DD} - \Pi_2^{DI} \\
&= \frac{B((\beta-1)c+\mu)^2}{2} \\
&\times \left(\frac{2\left(8B-\Delta^2\right)}{\left((6\beta-8)B-(\beta-1)\Delta^2\right)^2} - \frac{\left(4B-\Delta^2\right)\left((6\beta+8)B-(\beta+1)\Delta^2\right)^2}{\left(\left(\beta^2-1\right)\Delta^4+2\left(9\beta^2-16\right)B^2+3\left(4-3\beta^2\right)B\Delta^2\right)^2} \right) \\
&= \frac{B((\beta-1)c+\mu)^2}{2} \\
&\times \left(\frac{B^5 n\left(\beta^4\left(-(n-6)^2\right)\left((n-12)n+30\right)+2\beta^2(n-8)^2(n-6)(n-4)-(n-8)^3(n-4)\right)}{\left((6\beta-8)B-(\beta-1)\Delta^2\right)^2\left(\left(\beta^2-1\right)\Delta^4+2\left(9\beta^2-16\right)B^2+3\left(4-3\beta^2\right)B\Delta^2\right)^2} \right)
\end{aligned}$$

The second equality is derived by substituting $\Delta^2 = nB$. Excluding the terms which are always positive, the major term could be written as a quadratic function of $\phi = \beta^2$:

$$M(\phi) = -(n-6)^2((n-12)n+30)\phi^2 + 2(n-6)(n-4)(n-8)^2\phi - (n-8)^3(n-4)$$

The parabola opens down, has positive symmetric axis and negative intercept. Solving $M(\phi) = 0$ we get two roots:

$$\phi_1 = \frac{1}{n^4 - 24n^3 + 210n^2 - 792n + 1080}$$
$$\times\,(n^4 - 26n^3 + 248n^2 - \sqrt{2}\sqrt{n^6 - 40n^5 + 660n^4 - 5744n^3 + 27776n^2 - 70656n + 73728}$$
$$-\,1024n + 1536)$$

$$\phi_1 = \frac{1}{n^4 - 24n^3 + 210n^2 - 792n + 1080}$$
$$\times\,(n^4 - 26n^3 + 248n^2 + \sqrt{2}\sqrt{n^6 - 40n^5 + 660n^4 - 5744n^3 + 27776n^2 - 70656n + 73728}$$
$$-\,1024n + 1536)$$

When $6 - 2\sqrt{2} < n < 3.5$, $0 < \phi_1 < 1$; when $0 < n < 6 - 2\sqrt{2}$, $\phi_1 > 0$. ϕ_2 is always greater than 1. Therefore, let $\beta_1(n) = \sqrt{\phi_1}$ when $6-2\sqrt{2} < n < 3.5$ and $\beta_1(n) < \beta < 1$, $M(\phi) > 0$, i.e. $\Pi_2^{DD} > \Pi_2^{DI}$, $(Direct, Direct)$ is NE.

(2) Suppose (*Indirect, Indirect*) is the Nash equilibrium, then when manufacturer 1 chooses indirect collection, it is optimal for player 2 to choose indirect collection, that is, $\Pi_2^{II} > \Pi_2^{ID}$. When manufacturer 2 chooses indirect collection, it is optimal for player 2 to choose indirect collection, that is, $\Pi_1^{II} > \Pi_1^{DI}$. Also we have $\Pi_1^{II} = \Pi_2^{II}$ and

$\Pi_1^{DI} = \Pi_2^{ID}$. Therefore, the two inequalities are equivalent.

$$
\begin{aligned}
&\Pi_2^{II} - \Pi_2^{ID} \\
&= \frac{B((\beta-1)c+\mu)^2}{2} \\
&\times \left(\frac{4B-\Delta^2}{((3\beta-4)B-(\beta-1)\Delta^2)^2} - \frac{2\left(8B-\Delta^2\right)\left((3\beta+4)B-(\beta+1)\Delta^2\right)^2}{\left(\left(\beta^2-1\right)\Delta^4+2\left(9\beta^2-16\right)B^2+3\left(4-3\beta^2\right)B\Delta^2\right)^2} \right) \\
&= \frac{B((\beta-1)c+\mu)^2}{2} \\
&\times \left(\frac{B^5 n\left(\beta^4(n-3)^2((n-12)n+30)-2\beta^2(n-8)(n-4)^2(n-3)+(n-8)(n-4)^3\right)}{((3\beta-4)B-(\beta-1)\Delta^2)^2\left(\left(\beta^2-1\right)\Delta^4+2\left(9\beta^2-16\right)B^2+3\left(4-3\beta^2\right)B\Delta^2\right)^2} \right)
\end{aligned}
$$

Excluding the terms which are always positive, the major term could be written as a quadratic function of $\phi = \beta^2$:

$$
M_2(\phi) = (n-3)^2((n-12)n+30)\phi^2 - 2(n-8)(n-3)(n-4)^2\phi + (n-8)(n-4)^3
$$

When $0 < n < 3$, the parabola opens up, has positive symmetric axis and positive intercept.

When $n = 3$, $M(\phi) = 5$, always positive.

When $3 < n < 3.5$, the parabola opens up, has negative symmetric axis and positive intercept.

When $n \neq 3$, solving $M(\phi) = 0$ we get:

$$
\begin{aligned}
\phi_3 &= \frac{1}{n^4-18n^3+111n^2-288n+270} \\
&\times \left(n^4-19n^3+128n^2-\sqrt{2}\sqrt{n^6-26n^5+273n^4-1492n^3+4496n^2-7104n+4608}\right. \\
&\left.-368n+384\right)
\end{aligned}
$$

$$\phi_3 = \frac{1}{n^4 - 18n^3 + 111n^2 - 288n + 270}$$
$$\times (n^4 - 19n^3 + 128n^2 + \sqrt{2}\sqrt{n^6 - 26n^5 + 273n^4 - 1492n^3 + 4496n^2 - 7104n + 4608}$$
$$- 368n + 384)$$

When $0 < n < 3$, $\phi_3 > 1$ and $\phi_4 > 1$. When $3 < n < 3.5$, $\phi_3 < 0$ and $\phi_4 < 0$. Therefore, $M(\phi)$ is always positive when $0 < n < 3.5$ and $0 < \beta < 1$, that is, $(Indirect, Indirect)$ is always NE.

(3) Following the similar logic, we can also exclude the possibility that (*Direct, Indirect*) or (*Indirect, Direct*) might be Nash equilibrium. For example, if (*Direct, Indirect*) is Nash equilibrium, then the two inequalities are hold: (i) $\bar{\Pi}_2^{DD} < \bar{\Pi}_2^{DI}$, (ii) $\bar{\Pi}_1^{II} < \bar{\Pi}_1^{DI}$. That is to say, only when both (*Direct Direct*) and (*Indirect, Indirect*) are NOT equilibrium could (*Direct, Indirect*) be Nash equilibrium. This is not possible because (*Indirect, Indirect*) is Nash equilibrium in the available range.

Proof of Proposition 4.4

(1) If $(Direct, Direct)$ is Pareto efficient, then none of the following conditions should hold: (i) $(\bar{\Pi}_1^{DD}, \bar{\Pi}_2^{DD}) \preceq (\bar{\Pi}_1^{DI}, \bar{\Pi}_2^{DI})$, (ii) $(\bar{\Pi}_1^{DD}, \bar{\Pi}_2^{DD}) \preceq (\bar{\Pi}_1^{ID}, \bar{\Pi}_2^{ID})$, (iii) $(\bar{\Pi}_1^{DD}, \bar{\Pi}_2^{DD}) \preceq (\bar{\Pi}_1^{II}, \bar{\Pi}_2^{II})$. The first two conditions are equivalent, we only need to check (i) and (iii). For (i), when (*Dircect, Direct*) is the Nash equilibrium, we have $\bar{\Pi}_2^{DD} > \bar{\Pi}_2^{DI}$ hold. Therefore, condition (i) and (ii) can't be satisfied and (*Direct, Indirect*) can't be a Pareto improvement for (*Direct, Direct*). For (iii), we derive the condition from the

comparison of $\bar{\Pi}_1^{II}$ and $\bar{\Pi}_1^{DD}$.

$$\begin{aligned}
&\bar{\Pi}_1^{II} - \bar{\Pi}_1^{DD} \\
&= \frac{B((\beta-1)c+\mu)^2}{2}\left(\frac{4B-\Delta^2}{((3\beta-4)B-(\beta-1)\Delta^2)^2} + \frac{2\left(\Delta^2-8B\right)}{((6\beta-8)B-(\beta-1)\Delta^2)^2}\right) \\
&= \frac{B((\beta-1)c+\mu)^2}{2}\left(\frac{B^3 n\left(30\beta^2-64\beta+(\beta-1)^2n^2-12(\beta-1)^2n+32\right)}{((3\beta-4)B-(\beta-1)\Delta^2)^2\left((6\beta-8)B-(\beta-1)\Delta^2\right)^2}\right)
\end{aligned}$$

Excluding the terms which are always positive, the major term could be written as a quadratic function of β:

$$M(\beta) = \beta^2\left(n^2-12n+30\right) + \beta\left(-2n^2+24n-64\right) + n^2-12n+32$$

When $0 < n < 3.5$, the parabola opens up, has positive symmetric axis and positive intercept. Solving $M(\beta) = 0$, we get two roots:

$$root1 = \frac{n^2-\sqrt{2}\sqrt{n^2-12n+32}-12n+32}{n^2-12n+30}$$

$$root2 = \frac{n^2+\sqrt{2}\sqrt{n^2-12n+32}-12n+32}{n^2-12n+30}$$

When $0 < n < 3.5$, $0 < root1 < 1$ and $root2 > 1$. Therefore, when $0 < n < 3.5$ and $0 < \beta < root1$, $M(\beta)$ is positive, $\bar{\Pi}_1^{II} > \bar{\Pi}_1^{DD}$. When $0 < n < 3.5$ and $root1 < \beta < 1$, $\Pi_1^{II} < \Pi_1^{DD}$. Comparing $root1$ and $\beta_1(n)$, we can get $root1 < \beta < \beta_1(n)$. In the range when (*Direct, Direct*) is Nash equilibrium, condition (iii) does not hold. Therefore, we can conclude that $(Direct, Direct)$ is Pareto efficient if it is Nash Equilibrium.

(2) Suppose (*Indirect, Indirect*) is Pareto efficient, then none of the following three conditions should hold: (i) $(\bar{\Pi}_1^{II},\ \bar{\Pi}_2^{II}) \preceq (\bar{\Pi}_1^{DI},\ \bar{\Pi}_2^{DI})$, (ii) $(\bar{\Pi}_1^{II},\ \bar{\Pi}_2^{II}) \preceq (\bar{\Pi}_1^{ID},\ \bar{\Pi}_2^{ID})$, (iii) $(\bar{\Pi}_1^{II},\ \bar{\Pi}_2^{II}) \preceq (\bar{\Pi}_1^{DD},\ \bar{\Pi}_2^{DD})$. Again, the first two conditions are equivalent and when (*Indirect, Indirect*) is Nash equilibrium, we have $\bar{\Pi}_1^{II} > \bar{\Pi}_1^{DI}$. Condition (i) and (ii) do

not hold. From the comparison of $\bar{\Pi}_1^{II}$ and $\bar{\Pi}_1^{DD}$, we find that when $root1 < \beta < 1$, $\bar{\Pi}_1^{II} > \bar{\Pi}_1^{DD}$. Define $root1 = \beta_2(n)$, we can get (*Indirect, Indirect*) is Pareto efficient when $\beta_2(n) < \beta < 1$.

Proof of Proposition 4.7

The logic is similar with the proof of Proposition 1.

(1) Suppose (*Direct, Direct*) is the Nash equilibrium, then the payoff functions need to satisfy the following conditions: $\tilde{\Pi}_2^{DD} > \tilde{\Pi}_2^{DI}$ and $\tilde{\Pi}_1^{DD} > \tilde{\Pi}_1^{ID}$. The two inequalities are equivalent since $\tilde{\Pi}_1^{DD} = \tilde{\Pi}_2^{DD}$ and $\tilde{\Pi}_1^{ID} = \tilde{\Pi}_2^{DI}$. Therefore, we only need to compare $\tilde{\Pi}_2^{DD}$ and $\tilde{\Pi}_2^{DI}$.

$$
\begin{aligned}
&\tilde{\Pi}_2^{DD} - \tilde{\Pi}_2^{DI} \\
&= \frac{B(\mu-(1-\beta)c)^2}{4} \\
&\times \left(\frac{4B-\Delta^2}{((3\beta-4)B-(\beta-1)\Delta^2)^2} - \frac{16B\left((3\beta+4)B-(\beta+1)\Delta^2\right)^2}{\left(\left(\beta^2-1\right)\Delta^4+2\left(9\beta^2-16\right)B^2+3\left(4-3\beta^2\right)B\Delta^2\right)^2} \right)
\end{aligned}
$$

Reorganizing the terms and substitute $\Delta^2 = nB$, we can obtain that,

$$
\begin{aligned}
&\tilde{\Pi}_2^{DD} - \tilde{\Pi}_2^{DI} \\
&= \frac{nB^6(\mu-(1-\beta)c)^2}{4} \\
&\times \left(\frac{\beta^4\left(-(n-3)^2\right)\left(n^2-12\right)+2\beta^2(n-4)^2(n-3)(n+2)-(n-4)^3n}{((3\beta-4)B-(\beta-1)\Delta^2)^2(\left(\beta^2-1\right)\Delta^4+2\left(9\beta^2-16\right)B^2+3\left(4-3\beta^2\right)B\Delta^2)^2} \right)
\end{aligned}
$$

Excluding the terms which are always positive, the major term could be written as a quadratic function of $\phi = \beta^2$,

$$M(\phi) = -(n-3)^2\left(n^2-12\right)\phi^2 + 2(n-3)(n+2)(n-4)^2\phi - (n-4)^3n$$

Solving $M(\phi)=0$, we can obtain two roots:

$$root1 = \frac{n^4 - 9n^3 + 18n^2 - 4\sqrt{-n^5 + 18n^4 - 129n^3 + 460n^2 - 816n + 576} + 32n - 96}{n^4 - 6n^3 - 3n^2 + 72n - 108}$$

$$root2 = \frac{n^4 - 9n^3 + 18n^2 + 4\sqrt{-n^5 + 18n^4 - 129n^3 + 460n^2 - 816n + 576} + 32n - 96}{n^4 - 6n^3 - 3n^2 + 72n - 108}$$

The parabola has positive intercept and the two roots are all out of the range $(0,1)$. Therefore, when $0 < \phi < 1$, $M(\phi)$ is positive. $\tilde{\Pi}_2^{DD} > \tilde{\Pi}_2^{DI}$ always holds. (*Direct, Direct*) is always Nash equilibrium when $0 < \beta < 1$ and $0 < n < 3.5$.

(2) Suppose (*Indirect, Indirect*) is the Nash equilibrium, then the payoff functions need to satisfy the following conditions: $\tilde{\Pi}_2^{II} > \tilde{\Pi}_2^{ID}$ and $\tilde{\Pi}_1^{II} > \tilde{\Pi}_1^{DI}$. The two conditions are equivalent. We only need to check the first one. Substituting $\Delta^2 = nB$ into the equation and reorganize the terms, we can get:

$$\begin{aligned}
&\tilde{\Pi}_2^{II} - \tilde{\Pi}_2^{ID} \\
&= \frac{nB^6(\mu-(1-\beta)c)^2}{4} \\
&\times \left(\frac{\beta^4(n-6)^2\left(n^2-12\right) - 2\beta^2(n-8)(n-6)(n-4)(n+2) + (n-8)^2(n-4)n}{\left((6\beta-8)B-(\beta-1)\Delta^2\right)^2\left(\left(\beta^2-1\right)\Delta^4 + 2\left(9\beta^2-16\right)B^2 + 3\left(4-3\beta^2\right)B\Delta^2\right)^2} \right)
\end{aligned}$$

Excluding the terms which are always positive, the key term could be written as a quadratic function of $\phi = \beta^2$:

$$M(\phi) = (n-6)^2\left(n^2-12\right)\phi^2 - 2(n-6)(n-4)(n+2)(n-8)\phi + (n-4)n(n-8)^2$$

Solving $M(\phi)=0$, we can obtain two roots:

$$root1 = \frac{n^4 - 16n^3 + 68n^2 - 4\sqrt{-n^5 + 32n^4 - 404n^3 + 2512n^2 - 7680n + 9216} + 16n - 384}{n^4 - 12n^3 + 24n^2 + 144n - 432}$$

$$root2 = \frac{n^4 - 16n^3 + 68n^2 + 4\sqrt{-n^5 + 32n^4 - 404n^3 + 2512n^2 - 7680n + 9216} + 16n - 384}{n^4 - 12n^3 + 24n^2 + 144n - 432}$$

After checking the range of the roots and the shape of the parabola, we can find that when $2\sqrt{3} < n < 3.5$ and $\sqrt{root2} < \beta < 1$, $\tilde{\Pi}_2^{II} > \tilde{\Pi}_2^{ID}$. Define $\beta_3(n) = \sqrt{root2}$, we can conclude that when $2\sqrt{3} < n < 3.5$ and $\beta_2(n) < \beta < 1$, (*Indirect, Indirect*) is Nash equilibrium.

(3) We can also exclude the possibility that (*Direct, Indirect*) or (*Indirect, Direct*) might be Nash equilibrium. For example, if (*Direct, Indirect*) is Nash equilibrium, then the two inequalities are hold: (i) $\tilde{\Pi}_2^{DD} < \tilde{\Pi}_2^{DI}$, (ii) $\tilde{\Pi}_1^{II} < \tilde{\Pi}_1^{DI}$. That is to say, only when both (*Direct Direct*) and (*Indirect, Indirect*) are NOT equilibrium could (*Direct, Indirect*) be Nash equilibrium. This is not possible because (*Direct, Direct*) is Nash equilibrium in the whole available range.

Proof of Proposition 4.8

(1) If $(Direct, Direct)$ is Pareto efficient, then none of the following conditions should hold: (i) $(\tilde{\Pi}_1^{DD}, \tilde{\Pi}_2^{DD}) \preceq (\tilde{\Pi}_1^{DI}, \tilde{\Pi}_2^{DI})$, (ii) $(\tilde{\Pi}_1^{DD}, \tilde{\Pi}_2^{DD}) \preceq (\tilde{\Pi}_1^{ID}, \tilde{\Pi}_2^{ID})$, (iii) $(\tilde{\Pi}_1^{DD}, \tilde{\Pi}_2^{DD}) \preceq (\tilde{\Pi}_1^{II}, \tilde{\Pi}_2^{II})$. The first two conditions are equivalent, we only need to check (i) and (iii). For (i), when (*Dircect, Direct*) is the Nash equilibrium, we have $\bar{\Pi}_2^{DD} > \bar{\Pi}_2^{DI}$ hold. Therefore, condition (i) and (ii) can't be satisfied and (*Direct, Indirect*) can't be a Pareto improvement for (*Direct, Direct*). For (iii), we derive the condition from the comparison of $\tilde{\Pi}_1^{II}$ and $\tilde{\Pi}_1^{DD}$.

$$\begin{aligned}
&\tilde{\Pi}_1^{II} - \tilde{\Pi}_1^{DD} \\
&= \frac{B(\mu - (1-\beta)c)}{4} \times \left(\frac{(\beta-1)^2\Delta^6 + 4\beta(4-3\beta)B^2\Delta^2 + 4(\beta-1)B\Delta^4}{((3\beta-4)B - (\beta-1)\Delta^2)^2\,((6\beta-8)B - (\beta-1)\Delta^2)^2} \right) \\
&= \frac{nB^4(\mu - (1-\beta)c)}{4} \times \left(\frac{\beta^2(n^2-12) + \beta(-2n^2+4n+16) + n^2 - 4n}{((3\beta-4)B - (\beta-1)\Delta^2)^2\,((6\beta-8)B - (\beta-1)\Delta^2)^2} \right)
\end{aligned}$$

The second equality is derived by substitute $\Delta^2 = nB$ into the equation. The key term can be written as a quadratic function of β:

$$M(\beta) = \beta^2 \left(n^2 - 12\right) + \beta \left(-2n^2 + 4n + 16\right) + n^2 - 4n$$

Solving $M(\beta) = 0$, we can obtain two roots:

$$root1 = \frac{n^2 - 2n - 4\sqrt{4-n} - 8}{n^2 - 12}$$

$$root2 = \frac{n^2 - 2n + 4\sqrt{4-n} - 8}{n^2 - 12}$$

By checking the shape of the parabola and the range of two roots, we find that when $2\sqrt{3} < n < 3.5$ and $0 < \beta < \beta_4(n)$, $\tilde{\Pi}_1^{II} < \tilde{\Pi}_1^{DD}$, that is (*Direct, Direct*) is Pareto efficient. Note that $\beta_4(n) = root2$ here.

(2) Suppose (*Indirect, Indirect*) is Pareto efficient, then none of the following three conditions should hold (i) $(\tilde{\Pi}_1^{II},\ \tilde{\Pi}_2^{II}) \preceq (\tilde{\Pi}_1^{DI},\ \tilde{\Pi}_2^{DI})$, (ii) $(\tilde{\Pi}_1^{II},\ \tilde{\Pi}_2^{II}) \preceq (\tilde{\Pi}_1^{ID},\ \tilde{\Pi}_2^{ID})$, (iii) $(\tilde{\Pi}_1^{II},\ \tilde{\Pi}_2^{II}) \preceq (\tilde{\Pi}_1^{DD},\ \tilde{\Pi}_2^{DD})$. Condition (i) and (ii) could be guaranteed by the condition of Nash equilibrium for (*Indirect, Indirect*). For the comparison of $\tilde{\Pi}_1^{II}$ and $\tilde{\Pi}_1^{DD}$, we can find that (*Indirect, Indirect*) is Pareto efficient if it is Nash equilibrium.

Proof of Proposition 4.10

When manufacturer and retailer are engaged in vertical Nash, manufacturer's profit only depend on his own product recovery strategy. We can easily get $\hat{\Pi}_i^I > \hat{\Pi}_i^D$. Therefore, only (Indirect, Indirect) is Nash equilibrium. (Indirect, Indirect) is also Pareto efficient since no improvement could be identified such that one manufacturer's profit is improved while the other manufacturer's profit is not hurt.

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